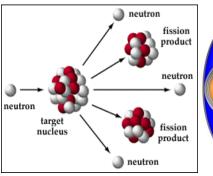


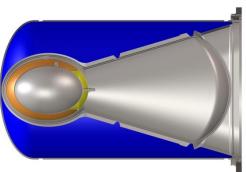
2017 NCNR Summer School on Methods and Applications of Neutron Spectroscopy

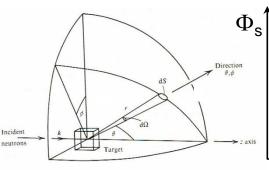


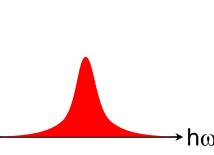
# Basic Elements of Neutron Inelastic Scattering

Peter M. Gehring
National Institute of Standards and Technology
NIST Center for Neutron Research
Gaithersburg, MD USA

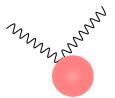




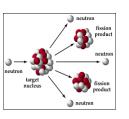




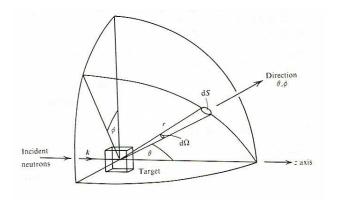
## **Outline**



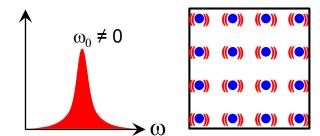
- 1. Introduction
  - Motivation
  - Scattering Probes



- 2. The Neutron
  - Production and Moderation
  - Wave/Particle Duality



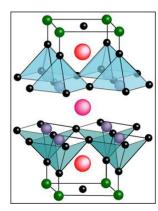
- 3. Basic Elements of Neutron Scattering
  - The Scattering Length b
  - Scattering Cross Sections
  - Pair Correlation Functions
  - Coherent and Incoherent Scattering
  - Neutron Scattering Methods



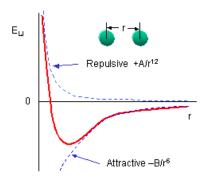
- 4. Summary of Scattering Cross Sections
  - Elastic (Bragg versus Diffuse)
  - Quasielastic (Diffusion)
  - Inelastic (Phonons)

## Structure and Dynamics

The most important property of any material is its underlying atomic / molecular structure (structure dictates function).



 $Bi_2Sr_2CaCu_2O_{8+\delta}$ 

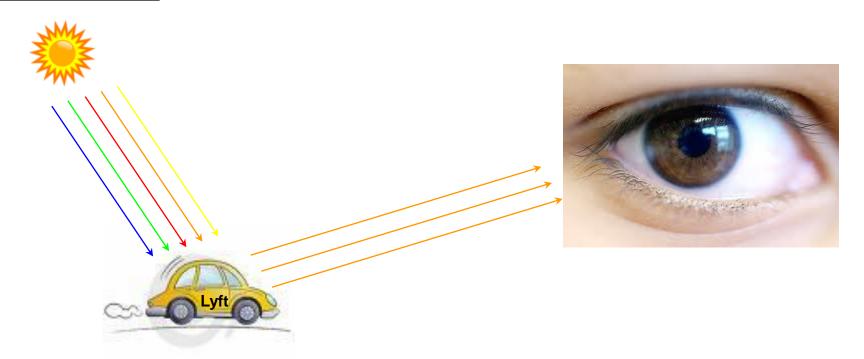


The motions of the atoms (dynamics) are extremely important because they provide information about the interatomic potentials.

An ideal method of characterization would provide detailed information about both structure and dynamics.

## Motivation

How do we "see"?



• We see something when light <u>scatters</u> from it.



Thus scattering conveys information!

Light is composed of electromagnetic <u>waves</u>.



 $\lambda \sim 4000 A - 7000 A$ 

However, the details of what we can see are ultimately <u>limited by the wavelength</u>.

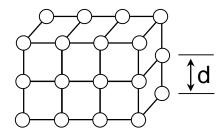
### **Motivation**



The tracks of a compact disk act as a diffraction grating, producing a separation of the colors of white light when it <u>scatters</u> from the surface.

From this one can determine the nominal distance between tracks on a CD, which is  $1.6 \times 10^{-6}$  meters = 16,000 Angstroms.

To characterize materials we must determine the <u>underlying structure</u>. We do this by using the material as a diffraction grating.



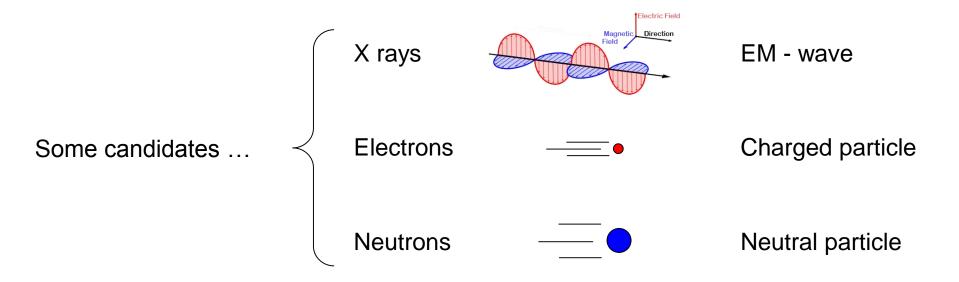
Problem: Distances between atoms in materials are of order Angstroms → light is inadequate.

Moreover, most materials are opaque to light.

$$\lambda_{Light} >> d \sim 4 \text{ Å}$$

# Scattering Probes

To measure atomic structure requires a probe with a  $\lambda$  ~ length scale of interest.



Remember de Broglie:  $\lambda = h/p = h/mv$ Particles have wave properties too.



Pros and Cons ...

Which one should we choose?

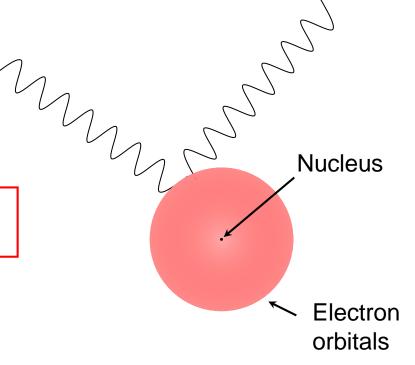
If we wish only to determine relative atomic positions, then we should choose x rays almost every time.

- 1. Relatively cheap
- 2. Sources are ubiquitous → easy access
- 3. High flux  $\rightarrow$  can study small samples

However ...

# Scattering Probes

X rays are electromagnetic radiation. Thus they scatter from the charge density.



## Consequences:

Low-Z elements are hard to see.

Elements with similar atomic numbers have very little contrast.

X rays are strongly attenuated as they pass through the walls of furnaces, cryostats, etc.

Hydrogen



(Z = 1)

Cobalt

**Nickel** 



??



$$(Z = 27)$$

$$(Z = 28)$$



Electrons are charged particles  $\rightarrow$  they see both the atomic electrons and nuclear protons at the same time.

- 1. Relatively cheap
- 2. Sources are not uncommon → easy access
- 3. Fluxes are extremely high  $\rightarrow$  can study tiny crystals
- 4. Very small wavelengths → more information

However ...

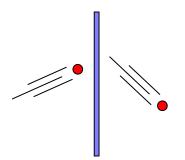
# Scattering Probes

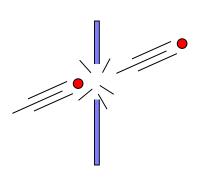
Electrons have some deficiencies too ...

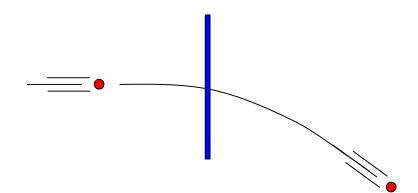
Requires very thin samples.

Radiation damage is a concern.

Magnetic structures are hard to determine because electrons are deflected by the internal magnetic fields.







# Scattering Probes

What about neutrons?



### Advantages

Wavelengths easily varied to match atomic spacings

Zero charge → not strongly attenuated by furnaces, etc.

Magnetic dipole moment → can study magnetic structures

Nuclear interaction  $\rightarrow$  can see low-Z elements easily like H  $\rightarrow$  good for the study of biomolecules and polymers.

Nuclear interaction is simple → scattering is easy to model

Low energies →
Non-destructive probe

### Disadvantages

Neutrons expensive to produce → access not as easy

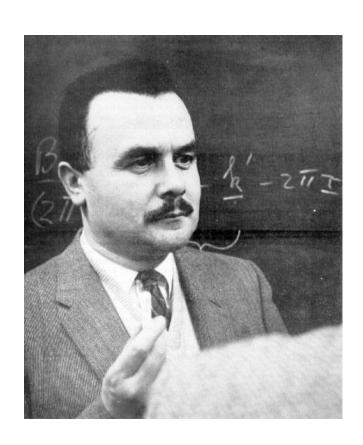
Interact weakly with matter

→ often require large samples

Available fluxes are low compared to those for x rays

Let's consider neutrons ...

## The Neutron



"If the neutron did not exist, it would need to be invented."

Bertram Brockhouse 1994 Nobel Laureate in Physics

# The Neutron



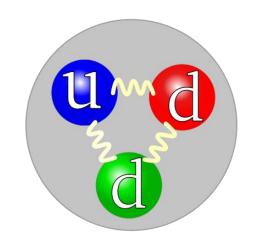
"... for the discovery of the neutron."

Sir James Chadwick 1935 Nobel Laureate in Physics

# The Neutron

$$m_n$$
= 1.675x10<sup>-27</sup> kg  
 $Q = 0$   
 $S = \frac{1}{2}$  h  
 $\mu_n$ = -1.913  $\mu_N$ 

1924: de Broglie Relation  $\lambda = h/p = h/m_n v$ 



$$\lambda = 1 \text{ Å}$$
  
 $v = 4000 \text{ m/s}$   
 $E = 82 \text{ meV}$ 

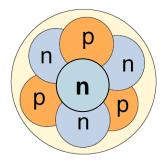
$$\lambda = 9 \text{ Å}$$
 $v = 440 \text{ m/s}$ 
 $E = 1 \text{ meV}$ 

## **Production**

Free neutrons decay via the weak force. Lifetime ~ 888 seconds (15 minutes).

$$n \rightarrow p + e^{-} + v_{e}$$

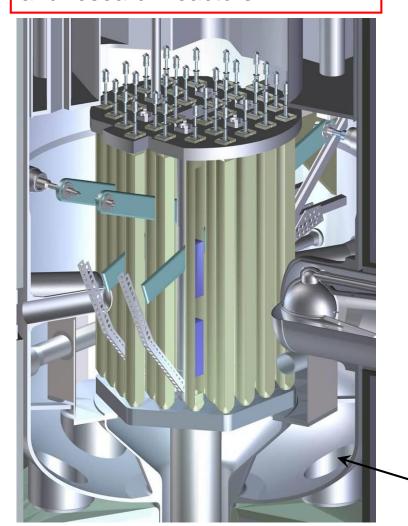
A useful source of neutrons requires a nuclear process by which bound neutrons can be freed from the nuclei of atoms and that is easily sustainable.

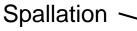


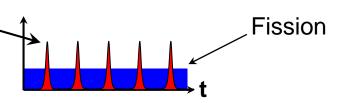
There are two such processes, spallation and fission ...

## **Fission**

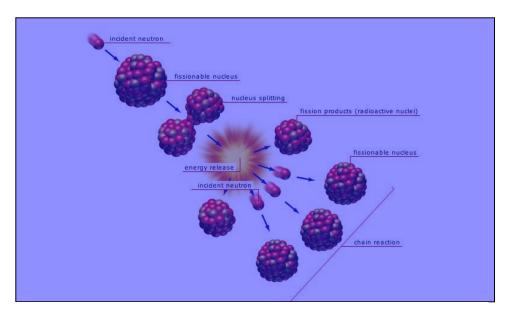
Nuclear fission is used in power and research reactors.







$${}^{235}_{92}U_{143} + n \rightarrow {}^{236}_{92}U_{144} \rightarrow X + Y + 2.44n$$

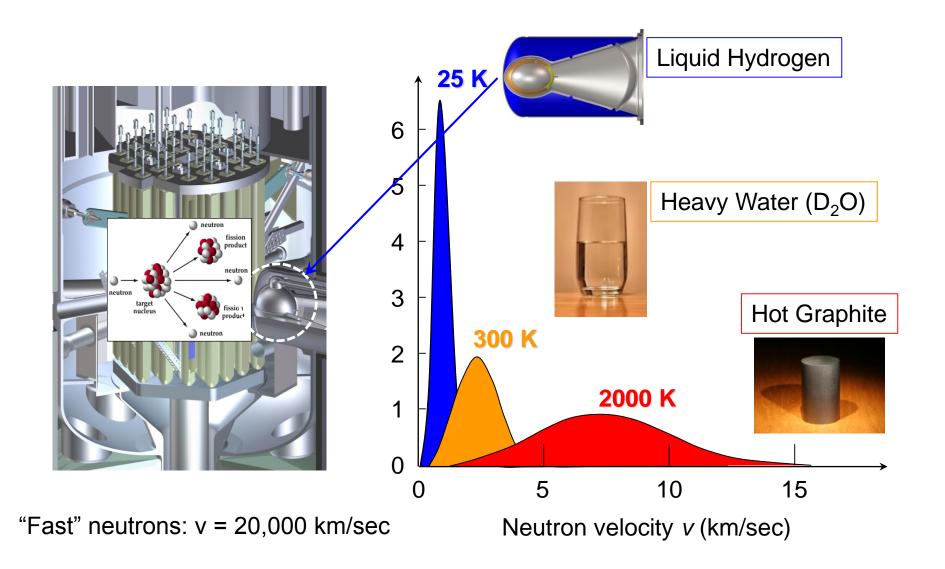


A liquid medium ( $D_2O$ , or heavy water) is used to moderate the fast fission neutrons to room temperature (2 MeV  $\rightarrow$  50 meV).

The fission process and moderator are confined by a large containment vessel.

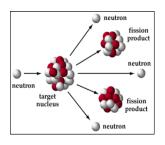
Maxwellian Distribution

$$\Phi \sim v^3 e^{(-mv^2/2k_BT)}$$

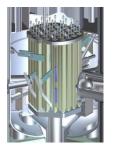


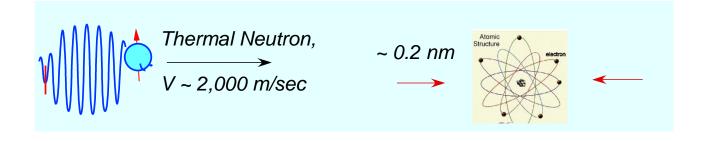
## Moderation

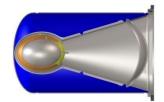
# de Broglie Relation $\lambda = h/m_n v$

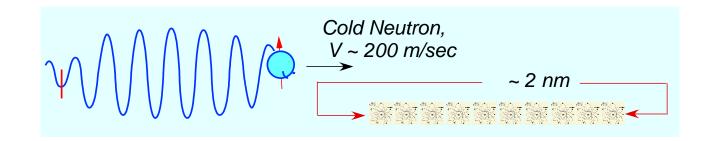












# Scattering Basics

Neutron scattering experiments measure the flux  $\Phi_s$  of neutrons scattered by a sample into a detector as a function of the change in neutron wave vector  $(\vec{Q})$  and energy  $(\hbar\omega)$ .

$$\begin{split} & \Phi_s(Q,\hbar\omega) = \frac{\text{neutrons}}{\text{sec-cm}^2} \\ & \frac{\text{Momentum}}{\hbar k} = \hbar(2\pi/\lambda) & \hbar\omega_n = \hbar^2 k_n^{\ 2}/2m \\ & \hbar \vec{Q} = \hbar \vec{k}_i - \hbar \vec{k}_f & \hbar\omega = \hbar\omega_i - \hbar\omega_f \end{split}$$

The expressions for the scattered neutron flux  $\Phi_s$  involve the positions and motions of atomic nuclei or unpaired electron spins.

$$\Phi_{s} = \mathbf{F}\{\vec{r}_{i}(t), \, \vec{r}_{j}(t), \, \vec{S}_{i}(t), \, \vec{S}_{j}(t)\}$$

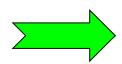


 $\Phi_s$  provides information about <u>all</u> of these quantities!

# Scattering Basics

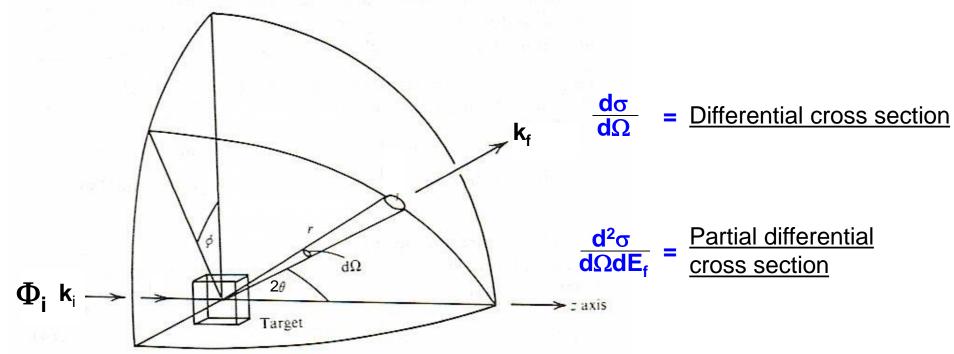
These "cross sections" are what we measure experimentally.

Consider an incident neutron beam with flux  $\Phi_i$  (neutrons/sec/cm²) and wave vector  $\mathbf{k}_i$  on a non-absorbing sample.



We define three cross sections:

**σ** = Total cross section



# Cross Sections

What are the physical meanings of these three cross sections?

σ

Total # of neutrons scattered per second /  $\Phi_i$ .

 $\frac{d\sigma}{d\Omega}$ 

Total # of neutrons scattered per second into d $\Omega$  / d $\Omega$   $\Phi_{\rm i}$ . (Diffraction  $\rightarrow$  structure.

 $\frac{d^2\sigma}{d\Omega dE_f}$ 

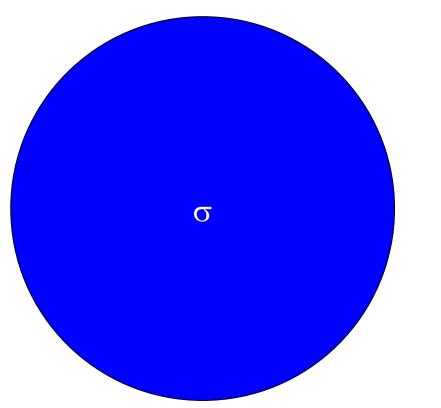
Total # of neutrons scattered per second into d $\Omega$  with a final energy between E<sub>f</sub> and dE<sub>f</sub> / d $\Omega$  dE<sub>f</sub>  $\Phi_i$ . (Inelastic scattering  $\rightarrow$  dynamics.

# Cross Sections

What are the relative sizes of the cross sections?

Clearly: 
$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d^2\sigma}{d\Omega dE_f} d\Omega dE_f$$

Thus: 
$$\sigma >> \frac{d\sigma}{d\Omega} >> \frac{d^2\sigma}{d\Omega dE_f}$$





$$\frac{d^2\sigma}{d\Omega dE_f}$$

Typically, 
$$\frac{d\sigma}{d\Omega} \sim \frac{10^6}{d\Omega} \times \frac{d^2\sigma}{d\Omega dE_f}$$

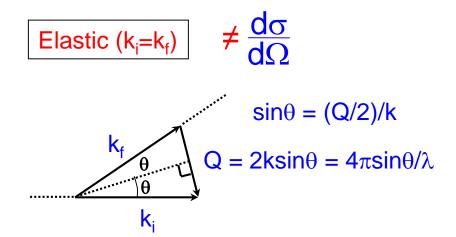
# Elastic vs Inelastic

Note that <u>both</u> of these cases are described by ...

$$\frac{d^2\sigma}{d\Omega dE_f}$$

### **Elastic Scattering:**

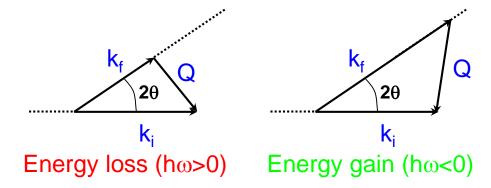
- Change in neutron energy = 0
- Probes changes in momentum only



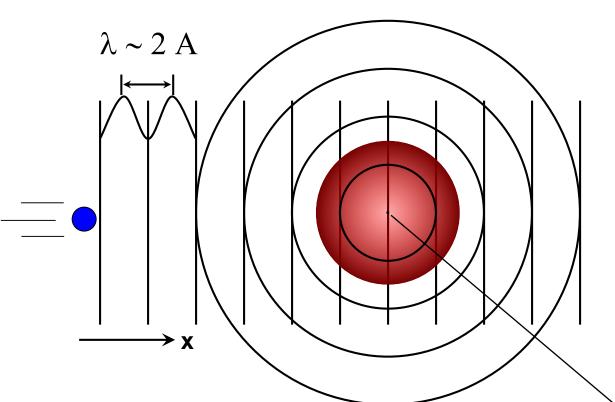
## **Inelastic Scattering:**

- Change in neutron energy ≠ 0
- Probes changes in <u>both</u> momentum and energy





Consider the simplest case: A fixed, isolated nucleus.



The scattered (final) neutron  $\Psi_f$  is a <u>spherical</u> wave:

$$\Psi_{\rm f}(\mathbf{r}) \sim (-b/r)e^{i\mathbf{k}\mathbf{r}}$$

### **QUESTIONS:**

- 1. The scattering is elastic  $(k_i = k_f = k)$ . Why?
- 2. The scattering is isotropic. Why?

The incident neutron  $\Psi_i$  is a <u>plane</u> wave:

$$\Psi_i(\mathbf{r}) \sim e^{i\mathbf{k}\mathbf{x}}$$

$$(k = 2\pi/\lambda)$$

Nucleus 1 fm = 
$$10^{-15}$$
 m

#### **ANSWERS:**

- 1. The scattering is elastic because the nucleus is fixed, so no energy can be transferred to it from the neutron (ignoring any excitations of the nucleus itself).
- 2. A basic result of diffraction theory states: if waves of any kind scatter from an object of a size  $<< \lambda$ , then the scattered waves are spherically symmetric. (This is also known as s-wave scattering.)

$$V(r) = \left(\frac{2\pi h^2}{m_n}\right) \sum_{j=1}^{N} b_j \delta(r-r_j)$$

Details of V(r) are unimportant! V(r) can be parametrized by a scalar b that depends only the nucleus and isotope!

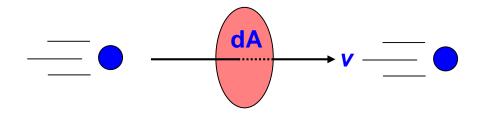
Fermi Pseudopotential

We can easily calculate  $\sigma$ for a single, fixed nucleus:

Def.

 $\sigma \cdot \Phi_i$  = Total number of neutrons scattered per second by the nucleus.

Velocity of neutrons (elastic → same before and after scattering).



 $V dA |\Psi_f|^2$  = Total number of neutrons scattered per second through dA.

$$\int V dA |\Psi_f|^2 = \int V(r^2 d\Omega)(b/r)^2 = \int V b^2 d\Omega = V 4\pi b^2 = \sigma \cdot \Phi_i$$

Since 
$$\Phi_i = V |\Psi_i|^2 = V \rightarrow \sigma = 4\pi b^2$$
 Try calculating

The Neutron Scattering Length - b

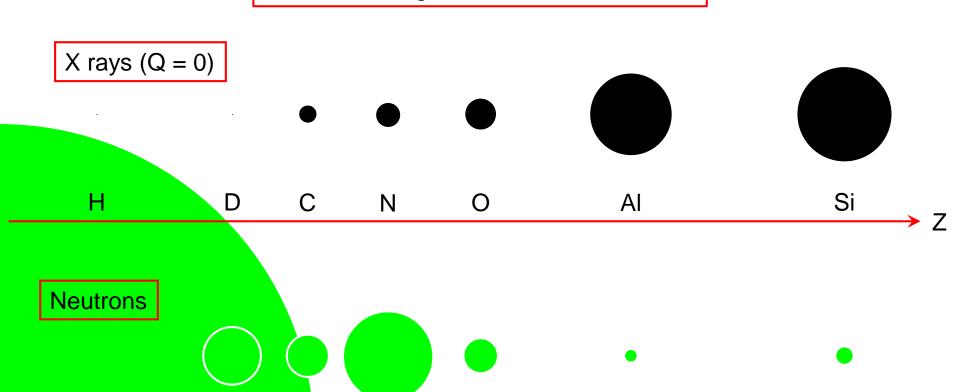
Units of length:  $b \sim 10^{-12}$  cm.

Analogous to f(Q), the x-ray scattering form factor.

Varies randomly with Z and isotope

→ Neutrons "see" atoms x rays can't.

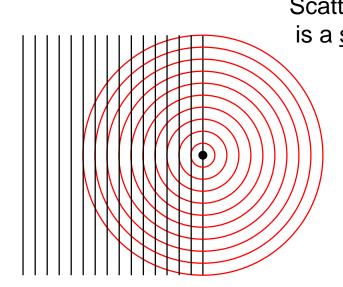
Total Scattering Cross Section:  $\sigma = 4\pi b^2$ 



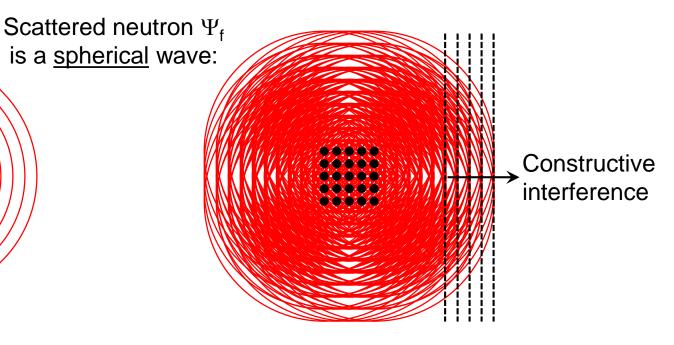
What if many atoms are present?

Scattering from one nucleus

Scattering from many nuclei



The incident neutron  $\Psi_i$  is a <u>plane</u> wave:



Get strong scattering in some directions, but not in others. Angular dependence yields information about how the nuclei are arranged or <u>correlated</u>.

# Magnetic Scattering

## What about magnetic scattering?

### **Nuclear Potential**

$$V_{N}(\mathbf{r}) = \frac{2\pi h^{2}}{m_{n}} b\delta(\mathbf{r})$$

Scalar interaction → Isotropic scattering

Very short range

Depends on nucleus, isotope, and nuclear spin

## **Magnetic Potential**

$$V_M(r) = -\mu_n \bullet B(r)$$

Vector interaction → Anisotropic scattering

Longer range

Depends on neutron spin orientation.

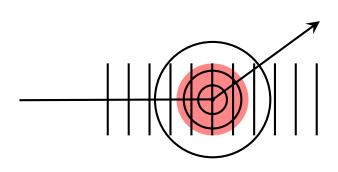


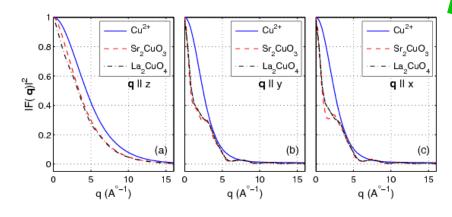
Polarized neutrons can measure the different components of M.

What about magnetic scattering?

### **Nuclear Potential**

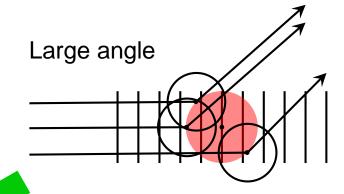
$$V_{N}(\mathbf{r}) = \frac{2\pi h^{2}}{m_{n}} b\delta(\mathbf{r})$$

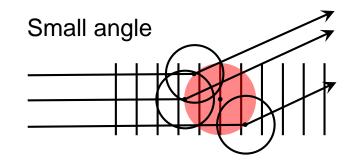




### **Magnetic Potential**

$$V_M(\mathbf{r}) = -\mu_n \bullet \mathbf{B}(\mathbf{r})$$





### (1) Born Approximation:

Assumes neutrons scatter only once (single scattering event).

## (2) Superposition:

Amplitudes of scattered neutrons  $\phi_n$  add linearly.

$$\Phi_s = \phi_1 + \phi_2 + \dots$$

Intensity = 
$$|\Phi|^2 = |\phi_1 + \phi_2 + \dots|^2 = |\phi_1|^2 + |\phi_2|^2 + \dots + |\phi_1^*\phi_2|^* + |\phi_2^*\phi_1| + \dots$$

Two-particle or pair correlations

After Andrew Boothroyd
PSI Summer School 2007

Depends on relative positions of 1 and 2  $\rightarrow$  pair correlations!

From Van Hove (1954) ...

The measured quantity  $\Phi_s$  depends only on time-dependent correlations between the positions of *pairs* of atoms.

This is true because <u>neutrons interact only weakly with matter</u>. Thus only the lowest order term in the perturbation expansion contributes.

#### **Differential Cross-Section**

$$\frac{d\sigma}{d\Omega} = \sum_{i,j} b_i b_j e^{-i(\mathbf{k}_i - \mathbf{k}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j)}$$

Depends only on:

where the atoms are

and

what the atoms are.

# From Squires (1996): Introduction to the theory of thermal neutron scattering

#### Partial Differential Cross-Section

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{k_0 \to k_1} = \frac{1}{N} \frac{k_f}{k_i} \left(\frac{m}{2\pi\hbar^2}\right)^2 \sum_{j=1}^{N} p_j \sum_{j=1}^{N} |\langle \mathbf{k}_f | V | \mathbf{k}_i \rangle|^2 \delta(E + E_i - E_f)$$

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{k_0 \to k_1} = N \frac{k_f}{k_i} b^2 S(\mathbf{Q}, \omega)$$

### **Neutron Structure Factor**

$$S(\mathbf{Q}, \omega) = \frac{1}{2\pi\hbar} \int G(\mathbf{r}, t) e^{i(\mathbf{Q} \cdot r - \omega t)} d\mathbf{r} dt$$

#### Pair Correlation Function

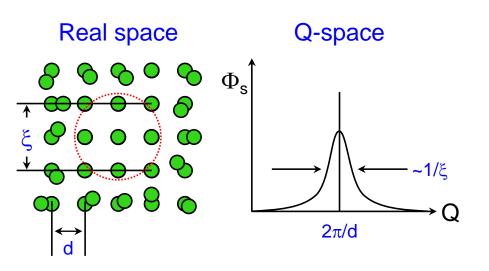
Fourier Transform

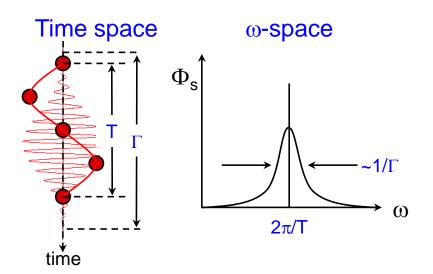
$$G(\mathbf{r},t) = \left(\frac{1}{2\pi}\right)^3 \frac{1}{N} \int \sum_{jj'} e^{i\mathbf{Q}\cdot\mathbf{r}} < e^{-i\mathbf{Q}\cdot\mathbf{r}_{j'}(0)} e^{i\mathbf{Q}\cdot\mathbf{r}_{j}(t)} > d\mathbf{Q}$$

**KEY IDEA** – Neutron interactions are <u>weak</u> → Scattering only probes <u>two-particle</u> correlations in space and time, but does so simultaneously!

The scattered neutron flux  $\Phi_s(\vec{Q},\hbar\omega)$  is proportional to the space  $(\vec{r})$  and time (t) Fourier transform of the <u>probability</u>  $G(\vec{r},t)$  of finding an atom at  $(\vec{r},t)$  given that there is another atom at r=0 at time t=0.

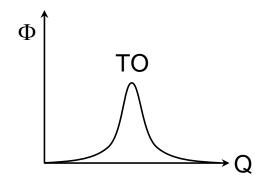
$$\Phi_{\mathbf{s}} \propto \frac{\partial^2 \sigma}{\partial \Omega \partial \omega} \propto \iint e^{i(\vec{Q}\cdot\vec{r}-\omega t)} G(\vec{r},t) d^3 \vec{r} dt$$



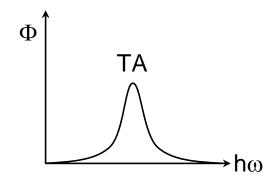


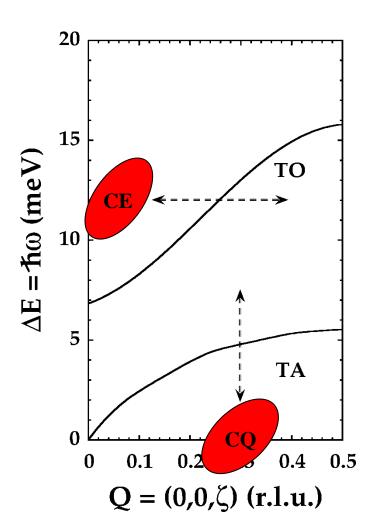
There are two main ways of measuring the neutron scattering cross section  $S(Q,\omega)$ .

Constant-E scans: vary Q at fixed hω.



Constant-Q scans: vary  $h\omega$  at fixed Q.





# Classic Example

## **BCS** Superconductivity

VOLUME 30, NUMBER 6

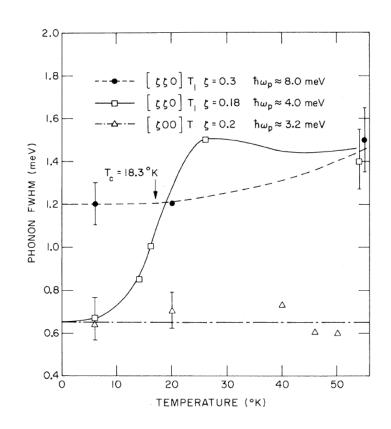
#### PHYSICAL REVIEW LETTERS

5 February 1973

#### Influence of the Superconducting Energy Gap on Phonon Linewidths in Nb<sub>3</sub>Sn†

J. D. Axe and G. Shirane

Brookhaven National Laboratory, Upton, New York 11973
(Received 7 December 1972)



### Coherent vs Incoherent

What happens when two different nuclei are randomly distributed throughout the crystal?

This situation could arise for two reasons.

- 1. Isotopic incoherence
- 2. Nuclear spin incoherence

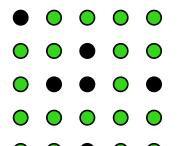
Both reasons can occur because the scattering interaction is nuclear.

Recall that 
$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{k_0\to k_1}=N\frac{k_f}{k_i}b^2S(\mathbf{Q},\omega)$$

Then the above equation must be generalized:

### Coherent vs Incoherent

What happens when two different nuclei are randomly distributed throughout the crystal?



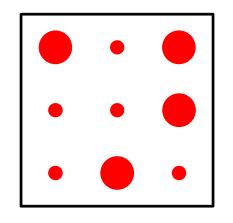
Our partial differential cross section can then be recast into the form:

$$\frac{d^2\sigma}{d\Omega dE_f} = \sigma_c S_c(Q,\omega) + \sigma_i S_i(Q,\omega) \quad \text{, where} \quad \label{eq:delta_delt$$

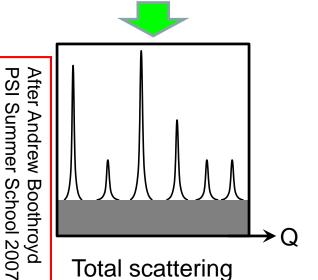
$$\sigma_c = 4\pi(\overline{b})^2 \qquad c = \text{coherent}$$
 
$$\sigma_i = 4\pi\{\overline{b^2} - (\overline{b})^2\} \qquad i = \text{incoherent}$$

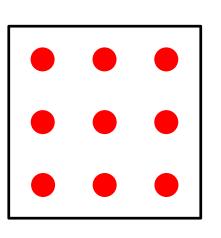
#### Coherent vs Incoherent

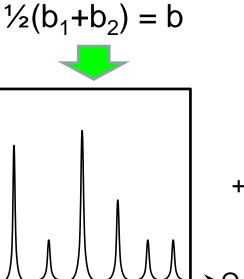
Consider a system composed of two different scattering lengths, b<sub>1</sub> and b<sub>2</sub>.

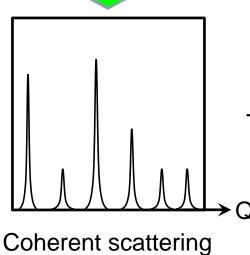


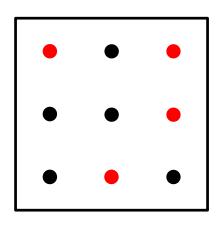
The two isotopes are randomly distributed.

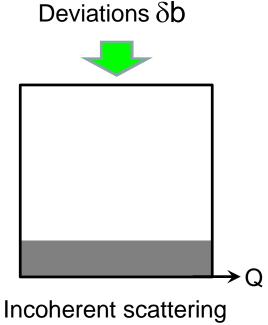












## Coherent vs Incoherent

What do these expressions mean physically?

**Coherent Scattering** 

**Incoherent Scattering** 

Measures the Fourier transform of the \*pair\* correlation function  $G(r,t) \rightarrow \underline{interference\ effects.}$ 

Measures the Fourier transform of the \*self\* correlation function  $G_s(r,t) \rightarrow no interference effects.$ 

This cross section reflects collective phenomena such as:

This cross section reflects single-particle scattering:

**Phonons** 

**Atomic Diffusion** 

Spin Waves

Vibrational Density of States

### Brief Summary

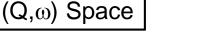
$$\frac{d\sigma}{d\Omega}\Big|_{coh} = \frac{\sigma_{coh}}{4\pi} S(Q)$$

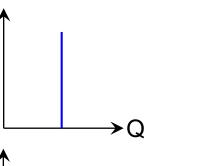
$$\frac{d\sigma}{d\Omega}\Big|_{inc} = \frac{\sigma_{inc}}{4\pi}$$

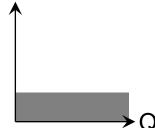
$$\frac{d^2\sigma}{d\Omega dE_f}\Big|_{coh} = \frac{k_f}{k_i} \frac{\sigma_{coh}}{4\pi} S_{coh}(Q,\omega)$$

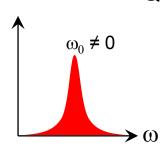
$$\frac{d^2\sigma}{d\Omega dE_f}\Big|_{inc} = \frac{k_f}{k_i} \frac{\sigma_{inc}}{4\pi} S_{inc}(Q,\omega)$$

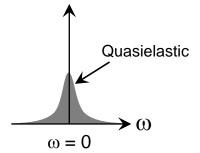




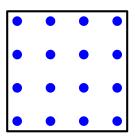


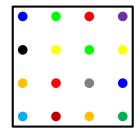


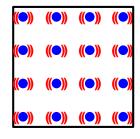




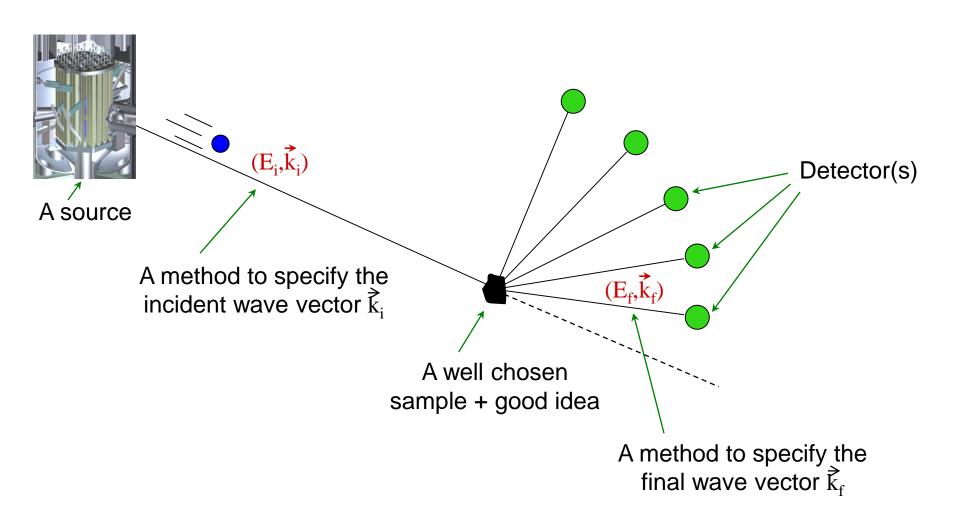




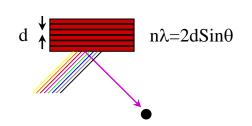




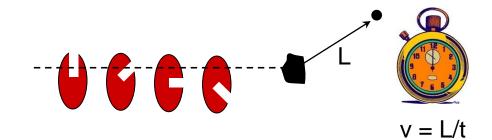




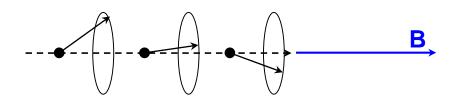
1. Bragg Diffraction



2. Time-of-Flight (TOF)

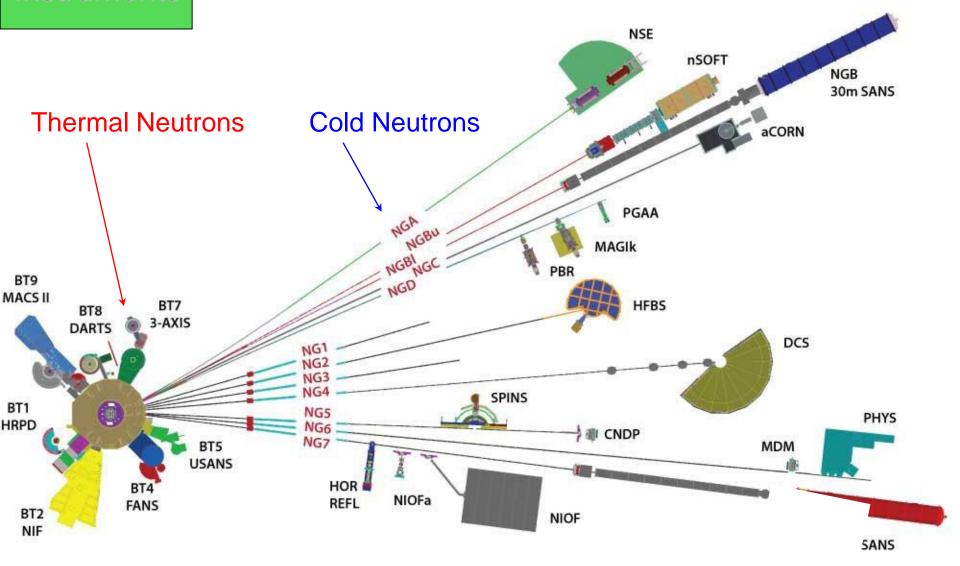


3. Larmor Precession



#### Neutron Instruments

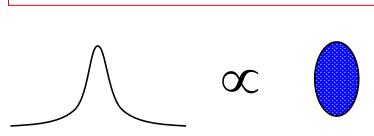
You will see many of these on the tour ...

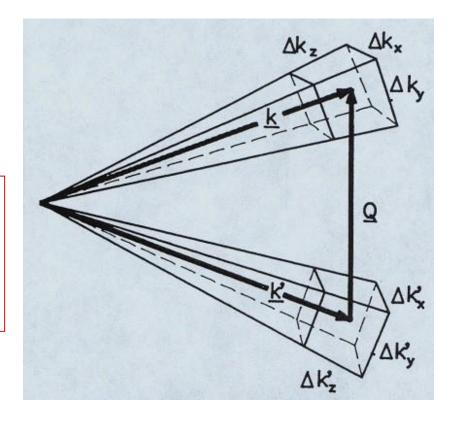


Because neutron scattering is an <u>intensity-limited</u> technique. Thus detector coverage and resolution MUST be tailored to the science.

Uncertainties in the neutron wavelength and direction imply  ${\bf Q}$  and  $\hbar\omega$  can only be defined with a finite precision.

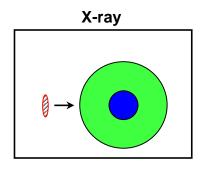
The total signal in a scattering experiment is proportional to the resolution volume → better resolution leads to lower count rates! Choose carefully ...

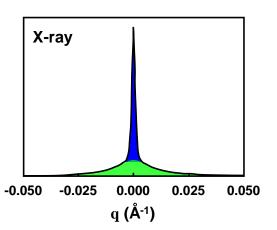


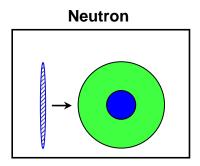


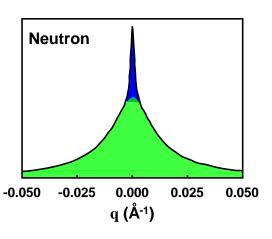
Courtesy of R. Pynn

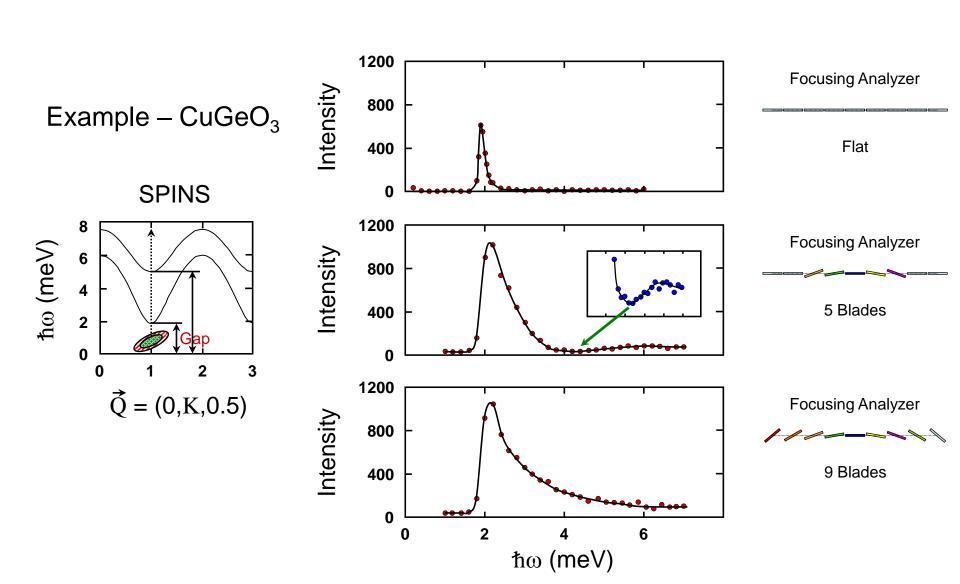
The "right" resolution depends on what you want to study.











### Quick Review

#### Please try to remember these things ...

- 1
- Neutrons scattering probes <u>two-particle</u> correlations in both space and time (simultaneously!).

2

The neutron scattering length, b, varies randomly with  $Z \rightarrow$  allows access to atoms that are usually unseen by x-rays.

(3)

**Coherent Scattering** 

4

**Incoherent Scattering** 

Measures the Fourier transform of the pair correlation function  $G(r,t) \rightarrow \underline{interference\ effects.}$ 

This cross section reflects collective phenomena.

Measures the Fourier transform of the self correlation function  $G_s(r,t) \rightarrow \underline{\text{no interference effects.}}$ 

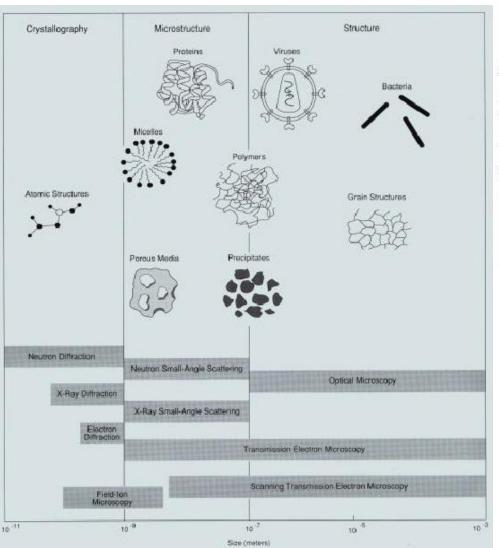
This cross section reflects single-particle scattering.

# More Examples

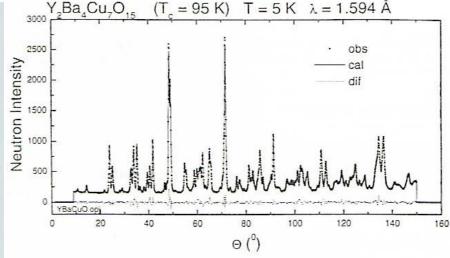
OK, after all of this, just exactly what can neutron scattering do for you?

Let's look at a few examples ...

# Elastic Scattering



Neutrons can probe length scales ranging from ~0.1 Å to ~1000 Å



Mitchell et. al, Vibrational Spectroscopy with Neutrons (2005)

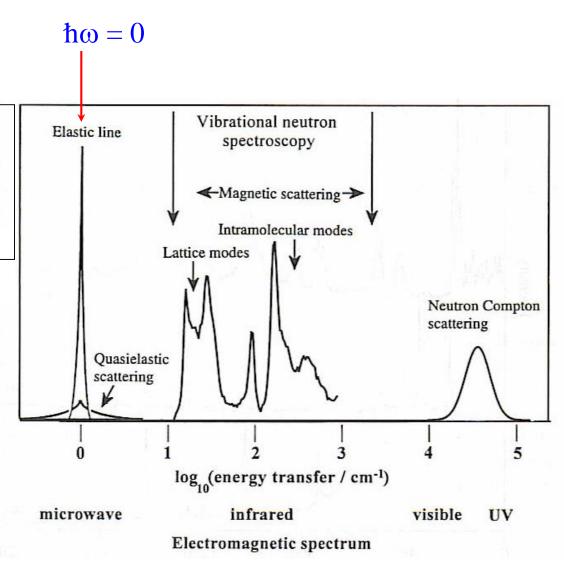
Neutrons needed to determine structure of 123 high-T<sub>c</sub> cuprates because x rays weren't sufficiently sensitive to the oxygen atoms.

Pynn, Neutron Scattering: A Primer (1989)

### Inelastic Scattering

Neutrons can probe time scales ranging from ~10<sup>-14</sup> s to ~10<sup>-8</sup>s.

Probes the vibrational, magnetic, and lattice excitations (dynamics) of materials by measuring changes in the neutron momentum and energy simultaneously.

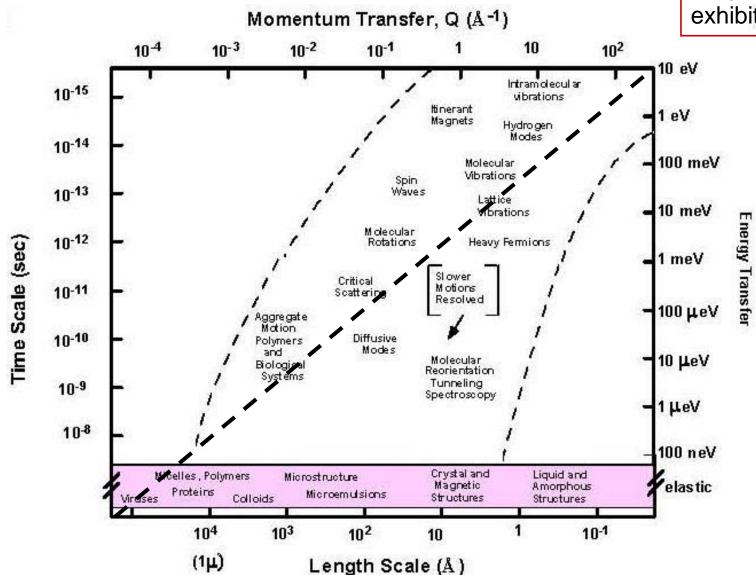


Mitchell et. al, Vibrational Spectroscopy with Neutrons (2005)

### Length and Time Scales

Do you see a pattern here?

Larger "objects" tend to exhibit slower motions.



# Elastic Scattering

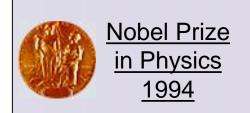
### **Pop Quiz**

Can one measure elastic scattering from a liquid?



If Yes, explain why?
If No, explain why not?

Hint: What is the correlation of one atom in a liquid with another after a time t?



The Fathers of Neutron Scattering

"For pioneering contributions to the development of neutron scattering techniques for studies of condensed matter"

"For the development of the neutron diffraction technique"



"For the development of neutron spectroscopy"



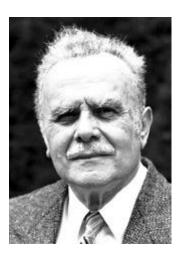
Clifford G Shull MIT, USA (1915 – 2001)

Showed us where the atoms are ...



Ernest O vvollar ORNL, USA (1910 – 1984)

Did first neutron diffraction expts ...



Bertram N Brockhouse McMaster University, Canada (1918 – 2003)

Showed us how the atoms move ...

### Useful References

- http://www.mrl.ucsb.edu/~pynn/primer.pdf
- "Introduction to the Theory of Thermal Neutron Scattering" G. L. Squires, Cambridge University Press
- "Theory of Neutron Scattering from Condensed Matter"- S. W. Lovesey, Oxford University Press
- "Neutron Diffraction" (Out of print)G. E. Bacon, Clarendon Press, Oxford
- "Structure and Dynamics"- M. T. Dove, Oxford University Press
- "Elementary Scattering Theory"- D. S. Sivia, Oxford University Press