Neutron Spin Echo

By

Roger Pynn*

Los Alamos National Laboratory
and University of California at Santa Barbara

*With contributions from B. Farago (ILL), S. Longeville (Saclay), and T. Keller (Stuttgart)
Neutron Spin Echo has significantly extended the (Q,E) range to which neutron scattering can be applied
The Underlying Physics of Neutron Spin Echo (NSE) Technology is Larmor Precession of the Neutron’s Spin

• The time evolution of the expectation value of the spin of a spin-1/2 particle in a magnetic field can be determined classically as:

\[ \frac{d\vec{s}}{dt} = \gamma \vec{s} \wedge \vec{B} \quad \Rightarrow \quad \omega_L = |\gamma| B \]

\[ \gamma = -2913 \times 2\pi \quad Gauss^{-1}.s^{-1} \]

• The total precession angle of the spin, \( \phi \), depends on the time the neutron spends in the field: \( \phi = \omega_L t \)

<table>
<thead>
<tr>
<th>B(Gauss)</th>
<th>( \omega_L ) ( (10^3 \text{ rad.s}^{-1}) )</th>
<th>N ( (\text{msec}^{-1}) )</th>
<th>Turns/m for 4 Å neutrons</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>183</td>
<td>29</td>
<td>~29</td>
</tr>
</tbody>
</table>
How does a Neutron Spin Behave when the Magnetic Field Changes Direction?

When \( H \) rotates with frequency \( \omega \), \( H_0 \rightarrow H_1 \rightarrow H_2 \ldots \), and the spin trajectory is described by a cone rolling on the plane in which \( H \) moves.

- **Distinguish two cases: adiabatic and sudden**
- **Adiabatic** – \( \tan(\delta) \ll 1 \) – large \( B \) or small \( \omega \) – spin and field remain co-linear – this limit used to “guide” a neutron spin
- **Sudden** – \( \tan(\delta) \gg 1 \) – large \( \omega \) – spin precesses around new field direction – this limit is used to design spin-turn devices
Larmor Precession allows the Neutron Spin to be Manipulated using Spin-Turn Coils ($\pi$ or $\pi/2$)

- The total precession angle of the spin, $\phi$, depends on the time the neutron spends in the B field

$$\phi = \omega_L t = \gamma B d / v$$

Number of turns = \(\frac{1}{135.65} B[\text{Gauss}] \cdot d[\text{cm}] \cdot \lambda[\text{Angstroms}]\)
Thin Magnetic Films used as $\pi/2$ and $\pi$ Rotators

$\phi = \frac{\gamma M d}{v \sin \chi}$

$H_g \rightarrow M \rightarrow \chi \rightarrow v$

$d \rightarrow M \rightarrow \chi \rightarrow v$

neutron
30 µ Permalloy (Ni$_{0.8}$ Fe$_{0.2}$) Films on Silicon Wafers used as Spin Turn Devices
In NSE\textsuperscript{*}, Neutron Spins Precess Before and After Scattering & a Polarization Echo is Obtained if Scattering is Elastic

Initially, neutrons are polarized along $z$. Rotate spins into x-y precession plane. Allow spins to precess around $z$: slower neutrons precess further over a fixed path length. Elastic Scattering Event. Rotate spins to $z$ and measure polarization.

Rotating spins through $\pi$ about $x$ axis.

\textbf{Final Polarization}, $P = \langle \cos(\phi_1 - \phi_2) \rangle$

* F. Mezei, Z. Physik, 255 (1972) 145
For Quasi-elastic Scattering, the Measured Neutron Polarization depends on Energy Transfer

- If the neutron changes energy when it scatters, the precession phases before & after scattering, $\phi_1$ & $\phi_2$, will be different:

\[
\phi_1 - \phi_2 = \gamma B d \left( 1 - \frac{1}{v_1} - \frac{1}{v_2} \right) \approx \frac{\gamma B d \hbar \omega}{m v^2} = \frac{\gamma B d m^2 \lambda^3 \omega}{2 \pi \hbar^2}
\]

- To lowest order, the difference between $\phi_1$ & $\phi_2$ depends only on $\omega$ (i.e. $v_1 - v_2$) & not on $v_1$ & $v_2$ separately

- The measured polarization, $\langle P \rangle$, is the average of $\cos(\phi_1 - \phi_2)$ over all transmitted neutrons i.e.

\[
\langle P \rangle = \frac{\iiint I(\lambda) S(\tilde{Q}, \omega) \cos(\phi_1 - \phi_2) d\lambda d\omega}{\iiint I(\lambda) S(\tilde{Q}, \omega) d\lambda d\omega}
\]
Neutron Polarization is Measured using an Assymetric Scan around the Echo Point

\[
\langle P \rangle = \frac{\int \int I(\lambda)S(\vec{Q}, \omega) \cos(\phi_1 - \phi_2) d\lambda d\omega}{\int \int I(\lambda)S(\vec{Q}, \omega) d\lambda d\omega} \approx \int S(\vec{Q}, \omega) \cos(\omega \tau) d\omega = I(\vec{Q}, t)
\]

where the "spin echo time" \( \tau = \gamma B d \frac{m^2}{2\pi h^2} \lambda^3 \)

<table>
<thead>
<tr>
<th>Bd (T.m)</th>
<th>( \lambda ) (nm)</th>
<th>( \tau ) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>0.6</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>186</td>
</tr>
</tbody>
</table>

The echo amplitude decreases when \((Bd)_1\) differs from \((Bd)_2\) because the incident neutron beam is not monochromatic.

For elastic scattering:

\[
\langle P \rangle \sim \int I(\lambda) \cos \left[ \frac{\gamma m}{h} \left\{ (Bd)_1 - (Bd)_2 \right\} \lambda \right] d\lambda
\]
What does a NSE Spectrometer Look Like?

IN11 at ILL was the First
Field-Integral Inhomogeneities cause $\tau$ to vary over the Neutron Beam: They can be Corrected

- Solenoids used as main precession fields have fields that vary as $r^2$ away from the axis of symmetry because of end effects ($\text{div } B = 0$)

- According to Ampere’s law, a current distribution that varies as $r^2$ can correct the field-integral inhomogeneities for parallel paths

- Similar devices can be used to correct the integral along divergent paths
Neutron Spin Echo study of Deformations of Spherical Droplets*

* Courtesy of B. Farago
The Principle of Neutron Resonant Spin Echo

- Within a coil, the neutron is subjected to a steady, strong field, $B_0$, and a weak rf field $B_1 \cos(\omega t)$ with a frequency $\omega = \omega_0 = \gamma B_0$
- Typically, $B_0 \sim 100$ G and $B_1 \sim 1$ G
- In a frame rotating with frequency $\omega_0$, the neutron spin sees a constant field of magnitude $B_1$
- The length of the coil region is chosen so that the neutron spin precesses around $B_1$ thru an angle $\pi$
- The neutron spin angle changes by $2\omega t_0 + \omega d/v$
Neutron Spin Phases in an NRSE Spectrometer*

Table 1. Spin orientation

<table>
<thead>
<tr>
<th>Time t</th>
<th>Phase field $B_r$</th>
<th>neutron Spin phase $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$t_A$</td>
<td>$\omega t_A$</td>
</tr>
<tr>
<td>A'</td>
<td>$t_{A'} = t_A + \frac{d}{v}$</td>
<td>$\omega t_{A'}$</td>
</tr>
<tr>
<td>B</td>
<td>$t_B = t_A + \frac{l_{AB} + d}{v}$</td>
<td>$\omega t_B$</td>
</tr>
<tr>
<td>B'</td>
<td>$t_{B'} = t_A + \frac{l_{AB} + 2d}{v}$</td>
<td>$\omega t_{B'}$</td>
</tr>
<tr>
<td>C</td>
<td>$t_C$</td>
<td>$-\omega t_C$</td>
</tr>
<tr>
<td>C'</td>
<td>$t_{C'} = t_C + \frac{d}{v}$</td>
<td>$-\omega t_{C'}$</td>
</tr>
<tr>
<td>D</td>
<td>$t_D = t_C + \frac{l_{CD} + d}{v}$</td>
<td>$-\omega t_D$</td>
</tr>
<tr>
<td>D'</td>
<td>$t_{D'} = t_C + \frac{l_{CD} + 2d}{v}$</td>
<td>$-\omega t_{D'}$</td>
</tr>
</tbody>
</table>

Echo occurs for elastic scattering when

$$l_{AB} + d = l_{CD} + d$$

* Courtesy of S. Longeville
The Measured Polarization for NRSE is given by an Expression Similar to that for Classical NSE

- Again, we assume that $v' = v + \delta v$ with $\delta v$ small and expand to lowest order, giving:

$$\langle P \rangle = \frac{\iiint I(\lambda)S(\vec{Q}, \omega) \cos(\omega \tau_{NRSE}) d\lambda d\omega}{\iiint I(\lambda)S(\vec{Q}, \omega) d\lambda d\omega}$$

where the "spin echo time" $\tau_{NRSE} = 2\gamma B_0 (l + d) \frac{m^2}{2\pi \hbar^2} \lambda^3$

- Note the additional factor of 2 in the echo time compared with classical NSE (a factor of 4 is obtained with "bootstrap" rf coils)
- The echo is obtained by varying the distance, $l$, between rf coils
- In NRSE, we measure neutron velocity using fixed "clocks" (the rf coils) whereas in NSE each neutron "carries its own clock" whose (Larmor) rate is set by the local magnetic field
An NRSE Triple Axis Spectrometer at HMI: Note the Tilted Coils
Measuring Line Shapes for Inelastic Scattering

- Spin echo polarization is the FT of scattering within the spectrometer transmission function.
- An echo is obtained when
  \[
  \frac{d}{dk_1} \left[ \frac{(Bd)_1}{k_1} - \frac{(Bd)_2}{k_2} \right] = 0 \quad \Rightarrow \quad \frac{N_1}{k_1^2} = \frac{N_2}{k_2^2}
  \]
- Normally the lines of constant spin-echo phase have no gradient in Q,\(\omega\) space because the phase depends only on \(|k|\).
- The phase lines can be tilted by using “tilted” precession magnets.
By “Tilting” the Precession-Field Region, Spin Precession Can Be Used to Code a Specific Component of the Neutron Wavevector

If a neutron passes through a rectangular field region at an angle, its total precession phase will depend only on $k_\perp$.

$$\omega_L = \gamma B$$

$$\phi = \omega_L t = \gamma B \frac{d}{v \sin \chi} = \frac{KBDd}{k_\perp}$$

with $K = 0.291$ (Gauss.cm.Å)$^{-1}$

Stop precession here

Start precession here
“Phonon Focusing”

- For a single incident neutron wavevector, $k_i$, neutrons are scattered to $k_F$ by a phonon of frequency $\omega_0$ and to $k_f$ by neighboring phonons lying on the “scattering surface”.
- The topology of the scattering surface is related to that of the phonon dispersion surface and it is locally flat.
- Provided the edges of the NSE precession field region are parallel to the scattering surface, all neutrons with scattering wavevectors on the scattering surface will have equal spin-echo phase.
“Tilted Fields”

- Phonon focusing using tilted fields is available at ILL and in Japan (JAERI)….however,
- The technique is more easily implemented using the NRSE method and is installed as an option on a 3-axis spectrometers at HMI and at Munich
- Tilted fields can also be used for elastic scattering and may be used in future to:
  - Increase the length scale accessible to SANS
  - Separate diffuse scattering from specular scattering in reflectometry
  - Measure in-plane order in thin films
  - Improve Q resolution for diffraction
An NRSE Triple Axis Spectrometer at HMI:
Note the Tilted Coils
Nanoscience & Biology Need Structural Probes for 1-1000 nm

10 nm holes in PMMA

CdSe nanoparticles

Peptide-amphiphile nanofiber

Actin

Si colloidal crystal

Structures over many length scales in self-assembly of ZnS and cloned viruses

Thin copolymer films
“Tilted” Fields for Diffraction: SANS

- Any unscattered neutron ($\theta=0$) experiences the same precession angles ($\phi_1$ and $\phi_2$) before and after scattering, whatever its angle of incidence.
- Precession angles are different for scattered neutrons.

\[
\phi_1 = \frac{KBD}{k \sin \chi} \quad \text{and} \quad \phi_2 = \frac{KBD}{k \sin(\chi + \theta)} \quad \Rightarrow \quad \cos(\phi_1 - \phi_2) \approx \cos \left( \frac{KBD \cos \chi}{k \sin^2 \chi} \theta \right)
\]

\[
P = \int dQ S(Q) \cos \left( \frac{KBD \cos \chi}{k^2 \sin^2 \chi} Q \right)
\]

Polarization proportional to Fourier Transform of $S(Q)$

Spin Echo Length, \[ r = \frac{KBD \cos \chi}{(k \sin \chi)^2} \]
How Large is the Spin Echo Length for SANS?

<table>
<thead>
<tr>
<th>Bd/sinχ (Gauss.cm)</th>
<th>λ (Angstroms)</th>
<th>χ (degrees)</th>
<th>r (Angstroms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,000</td>
<td>4</td>
<td>20</td>
<td>1,000</td>
</tr>
<tr>
<td>5,000</td>
<td>4</td>
<td>20</td>
<td>1,500</td>
</tr>
<tr>
<td>5,000</td>
<td>6</td>
<td>20</td>
<td>3,500</td>
</tr>
<tr>
<td>5,000</td>
<td>6</td>
<td>10</td>
<td>7,500</td>
</tr>
</tbody>
</table>

It is relatively straightforward to probe length scales of ~ 1 micron.
Using “Sign Reversal” to Implement Angle Coding

An element that performs a $\pi$ rotation about an axis in the precession plane changes the sign of prior precession angles.

Net Precession = $-\phi_1 + \phi_2 - \phi_3 + \phi_4 - \phi_5 + \phi_6$

- Total net precession angle = $(\phi_4 + \phi_5 + \phi_6) - (\phi_1 + \phi_2 + \phi_3) + 2(\phi_2 - \phi_5)$
- The first two terms depend on neutron velocity only, last term depends on velocity and angle of neutron trajectory
  - Requires suitably oriented planar $\pi$ rotators (flippers)
- Can be set up to encode two, mutually perpendicular, trajectory angles
Conclusion:
NSE Provides a Way to Separate Resolution from Monochromatization & Collimation

• The method currently provides the best energy resolution for inelastic neutron scattering (~ neV)
  – Both classical NSE and NRSE achieve similar energy resolution
  – NRSE is more easily adapted to “phonon focusing”

• The method is likely to be used in future to improve (Q) resolution for elastic scattering
  – Extend size range for SANS (“rescue” scattering from the beam-stop region)
  – May allow 100x gain in measurement speed for SANS
  – Separate specular and diffuse scattering in reflectometry
  – Measure in-plane ordering in thin films (Felcher)