Neutron Spin Echo

By

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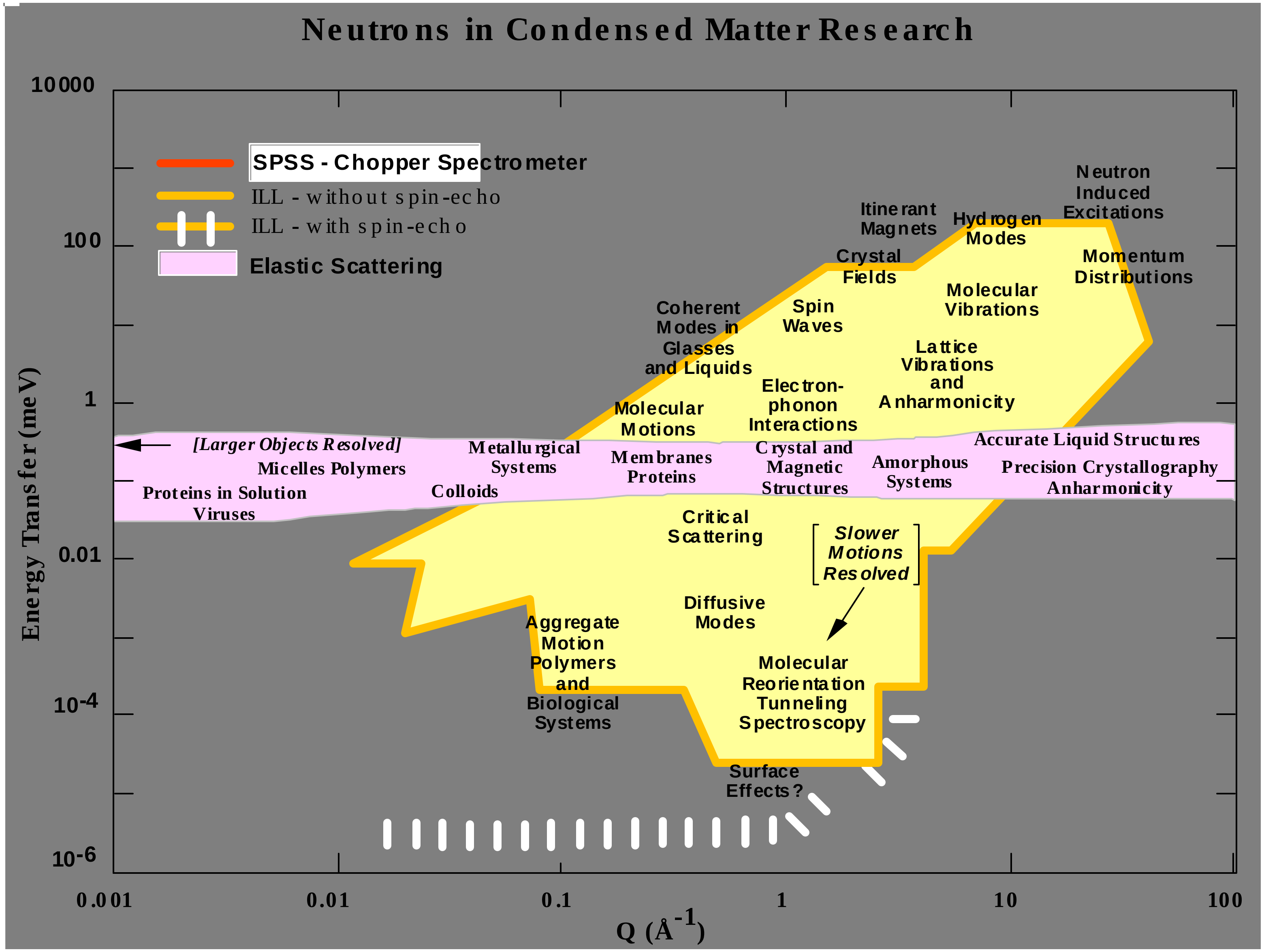
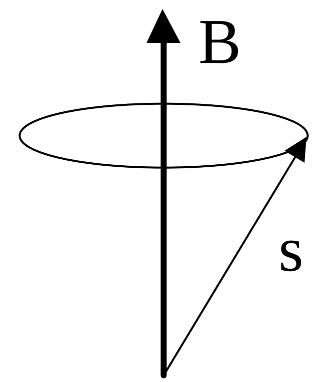


Neutron Spin Echo has significantly extended the (Q,E) range to which neutron scattering can be applied

The Underlying Physics of Neutron Spin Echo (NSE) Technology is Larmor Precession of the Neutron's Spin

- The time evolution of the expectation value of the spin of a spin-1/2 particle in a magnetic field can be determined classically as:



$$\frac{d\vec{s}}{dt} = g\vec{s} \wedge \vec{B} \qquad \Rightarrow \quad w_L = |g|B$$

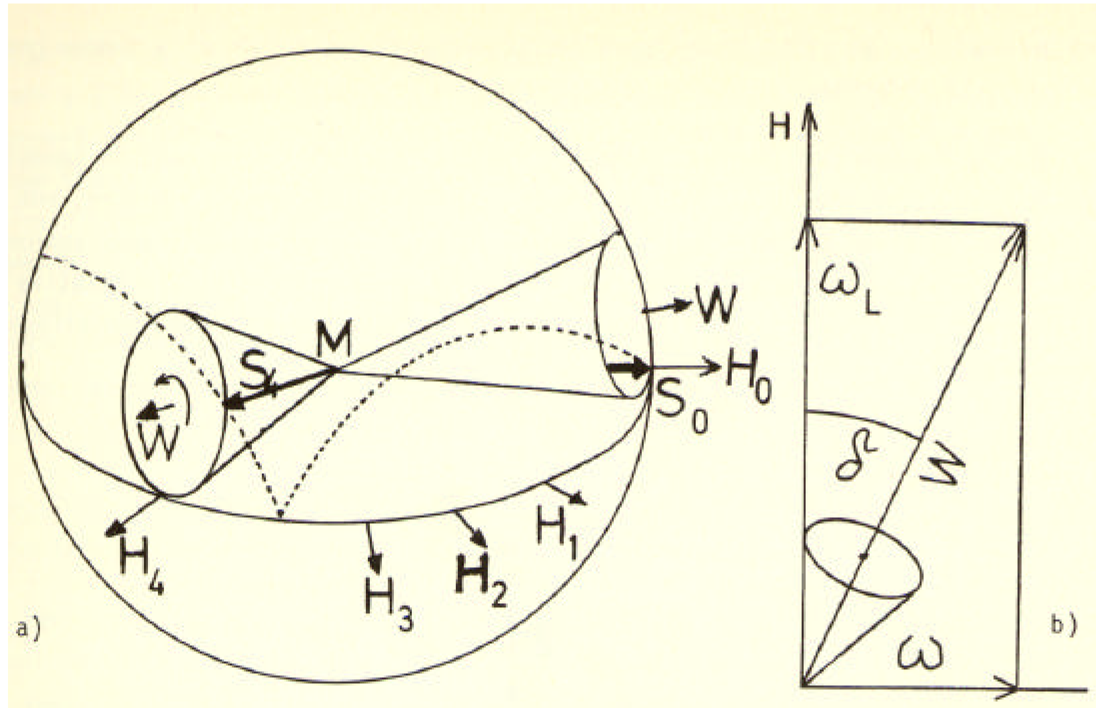
$$g = -2913 * 2p \quad Gauss^{-1}.s^{-1}$$

- The total precession angle of the spin, f, depends on the time the neutron spends in the field: $f = w_L t$

B(Gauss)	w_L (10^3 rad.s^{-1})	N ($msec^{-1}$)	Turns/m for 4 Å neutrons
10	183	29	~29

How does a Neutron Spin Behave when the Magnetic Field Changes Direction?

When H rotates with frequency ω, H_0->H_1 ->H_2…, and the spin trajectory is described by a cone rolling on the plane in which H moves

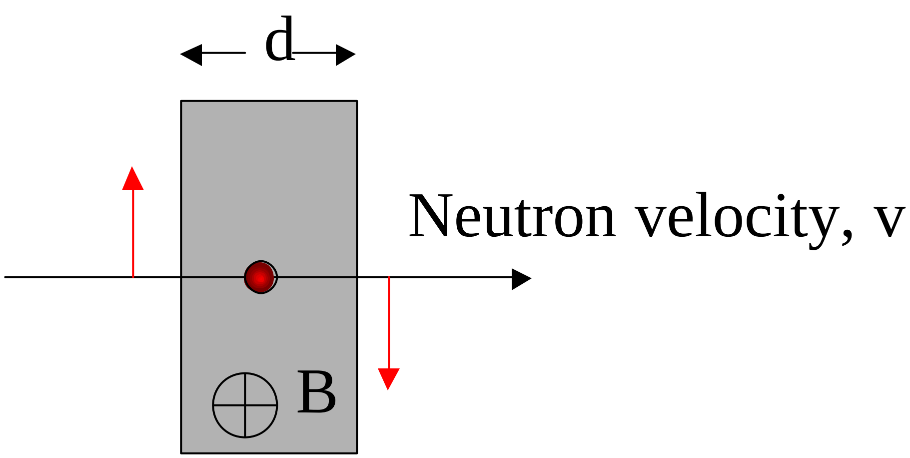


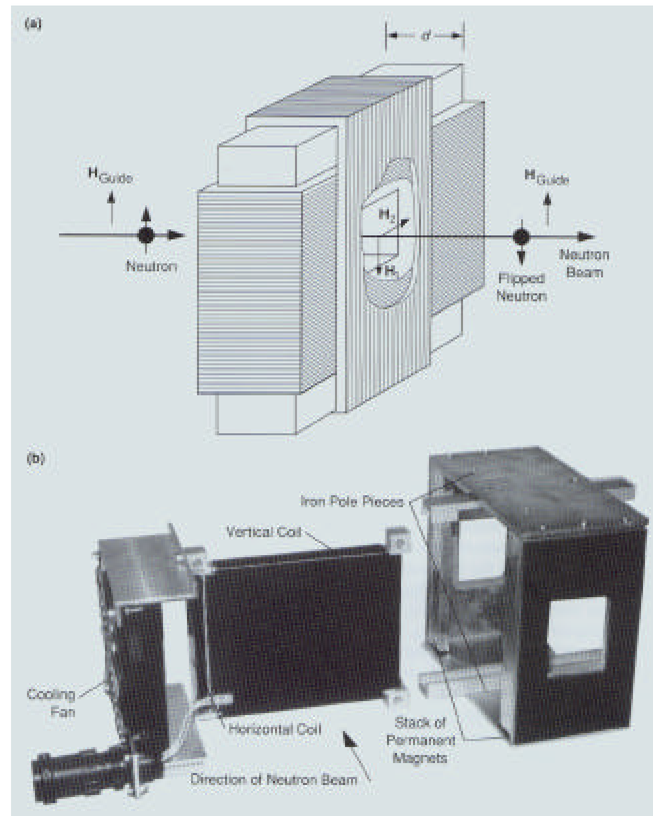
- Distinguish two cases: adiabatic and sudden
- *Adiabatic* – tan(δ) << 1 – large B or small ω – spin and field remain co-linear – this limit used to “guide” a neutron spin
- *Sudden* – tan(δ) >> 1 – large ω – spin precesses around new field direction – this limit is used to design spin-turn devices

Larmor Precession allows the Neutron Spin to be Manipulated using Spin-Turn Coils (π or π/2)

- The total precession angle of the spin, f, depends on the time the neutron spends in the B field

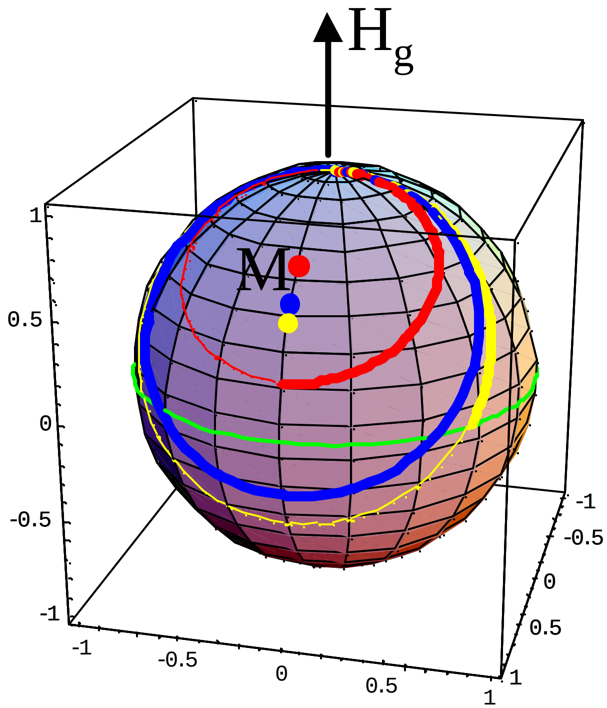
$$f = w_L t = gBd / \mathrm{v}$$

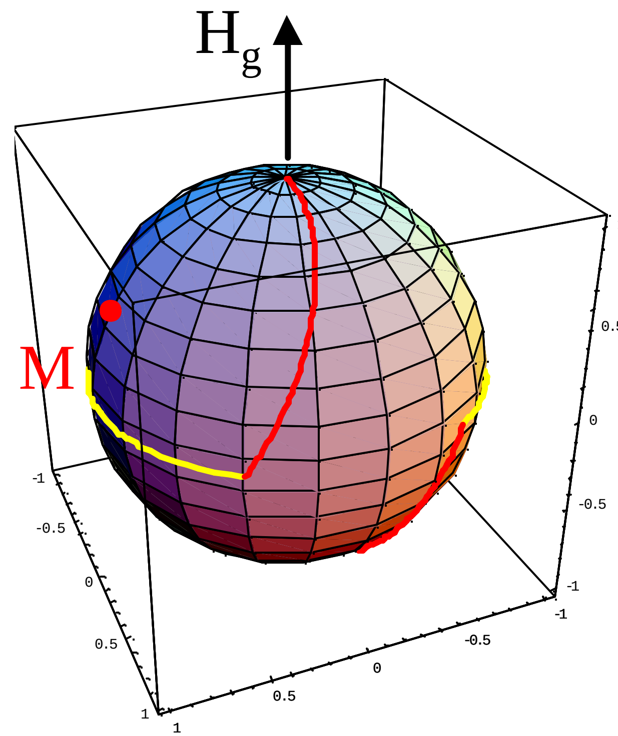




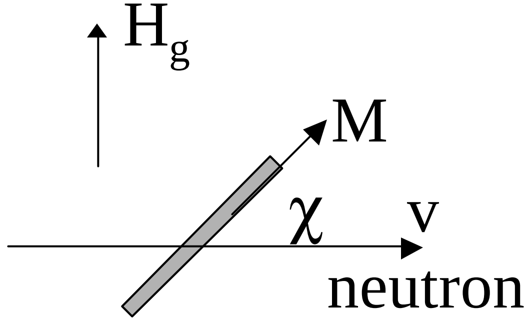
$$\text{Number of turns} = \frac{1}{135.65}.B[Gauss].d[cm].l[Angstroms]$$

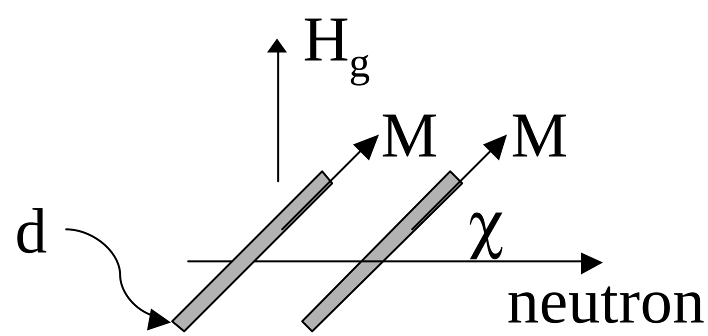
Thin Magnetic Films used as $\pi/2$ and π Rotators



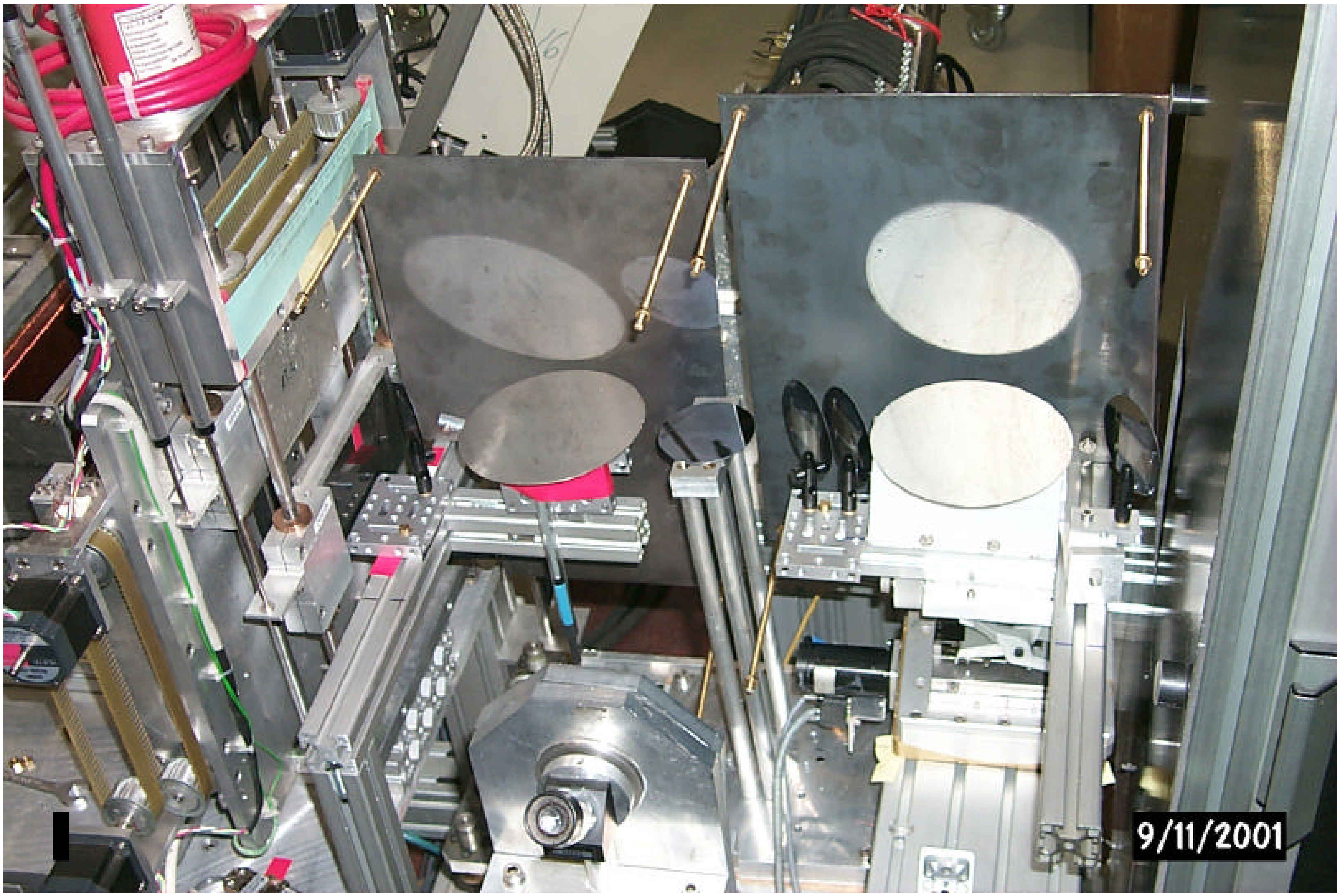


$$f = \frac{gMd}{v \sin c}$$

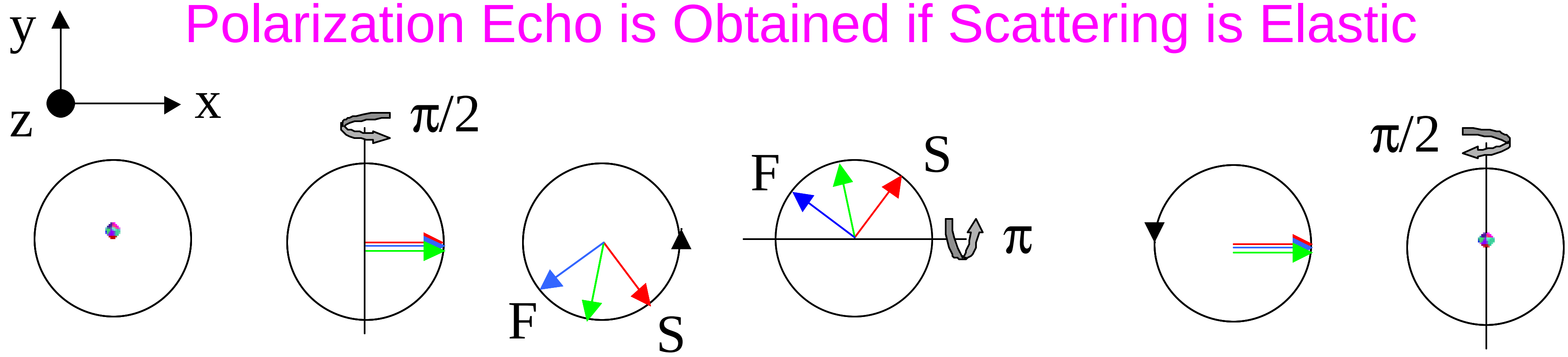




30 μ Permalloy ($Ni_{0.8}$ $Fe_{0.2}$) Films on Silicon Wafers used as Spin Turn Devices



In NSE*, Neutron Spins Precess Before and After Scattering & a Polarization Echo is Obtained if Scattering is Elastic



Initially, neutrons are polarized along z

Rotate spins into x-y precession plane

Allow spins to precess around z: slower neutrons precess further over a fixed path length

Rotate spins through π about x axis

Elastic Scattering Event

Allow spins to precess around z: precession angle of all neutrons is the same at echo point if ΔE = 0

Rotate spins to z and measure polarization

$$\boxed{\textit{Final Polarization}, P = \langle \cos(f_1 - f_2) \rangle}$$

* F. Mezei, Z. Physik, 255 (1972) 145

For Quasi-elastic Scattering, the Measured Neutron Polarization depends on Energy Transfer

- If the neutron changes energy when it scatters, the precession phases before & after scattering, f_1 & f_2, will be different:

$$\text{using} \qquad \hbar w = \frac{1}{2} m(v_1^2 - v_2^2) \approx mvdv$$

$$f_1 - f_2 = gBd\left(\frac{1}{v_1} - \frac{1}{v_2}\right) \approx \frac{gBd}{v^2}dv \approx \frac{gBd\hbar w}{mv^3} = \frac{gBdm^2 l^3 w}{2ph^2}$$

- To lowest order, the difference between f_1 & f_2 depends only on ω (I.e. $v_1 - v_2$) & *not* on v_1 & v_2 separately
- The measured polarization, <P>, is the average of $\cos(f_1 - f_2)$ over all transmitted neutrons I.e.

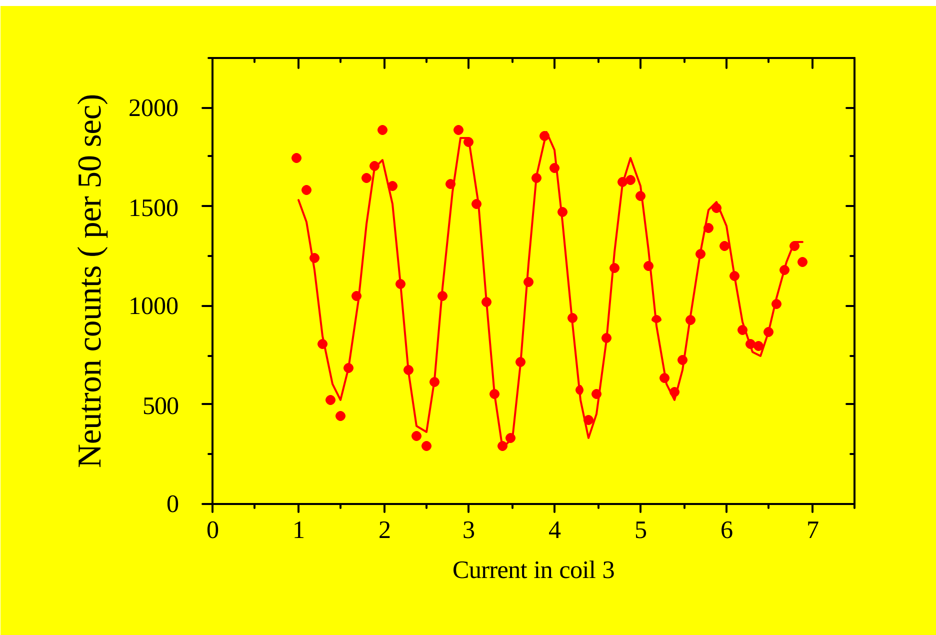
$$\langle P \rangle = \frac{\iint I(l) S(\vec{Q},w) \cos(f_1 - f_2) d l dw}{\iint I(l) S(\vec{Q},w) d l dw}$$

Neutron Polarization is Measured using an Assymetric Scan around the Echo Point

$$\langle P \rangle = \frac{\iint I(l)S(\vec{Q},w)\cos(f_1 - f_2)\,dl\,dw}{\iint I(l)S(\vec{Q},w)\,dl\,dw} \approx \left\langle \int S(\vec{Q},w)\cos(wt)\,dw \right\rangle = I(\vec{Q},t)$$

where the "spin echo time" $t = gBd\dfrac{m^2}{2ph^2}l^3$

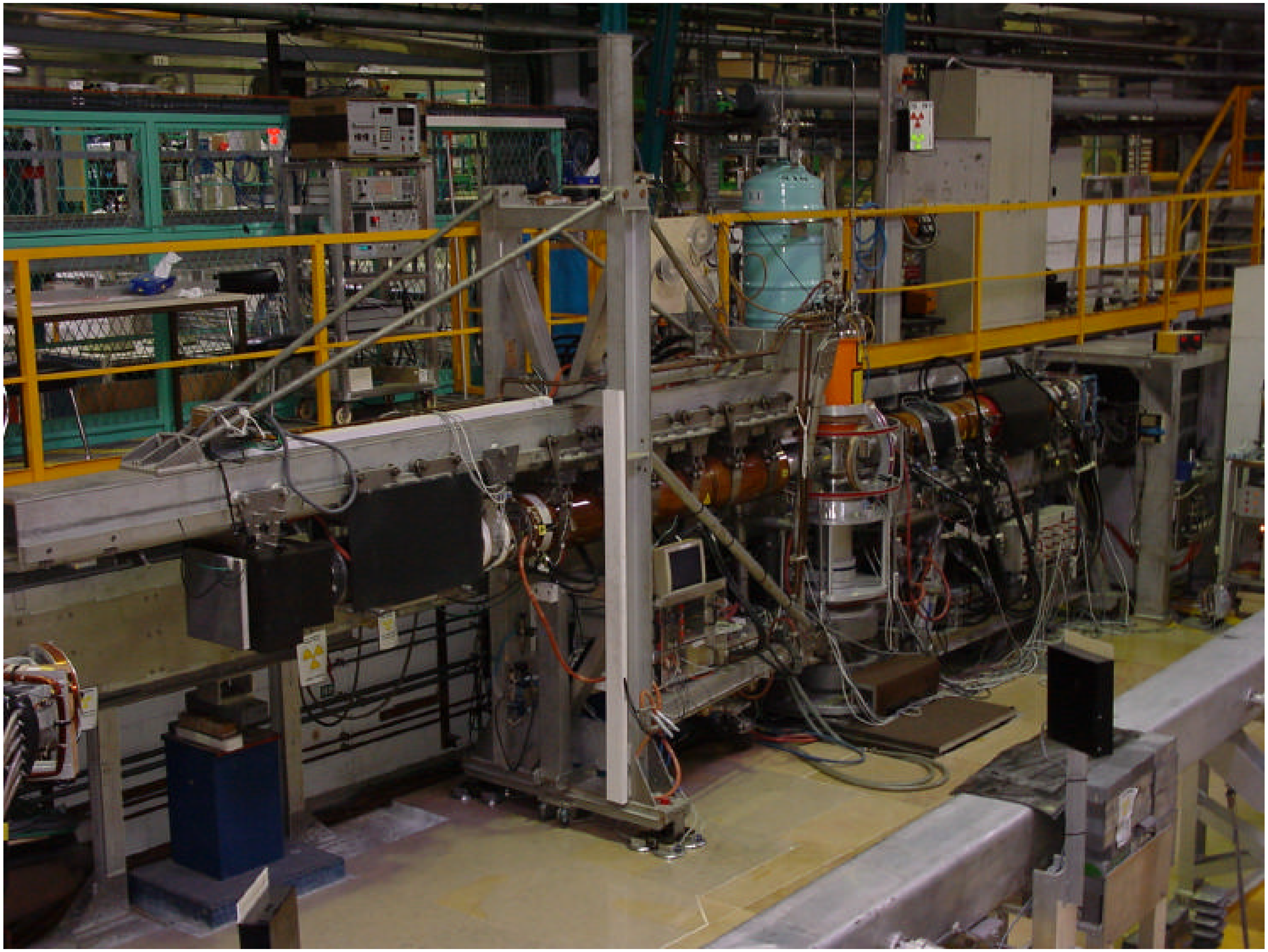
Bd (T.m)	λ (nm)	τ (ns)
1	0.4	12
1	0.6	40
1	1.0	186



The echo amplitude decreases when $(Bd)_1$ differs from $(Bd)_2$ because the incident neutron beam is not monochromatic. For elastic scattering:

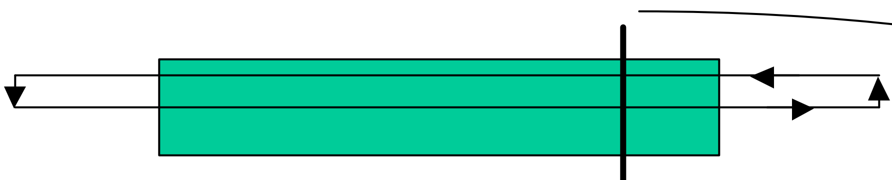
$$\langle P \rangle \sim \int I(l)\cos\left[\frac{gm}{h}\{(Bd)_1 - (Bd)_2\}l\right].dl$$

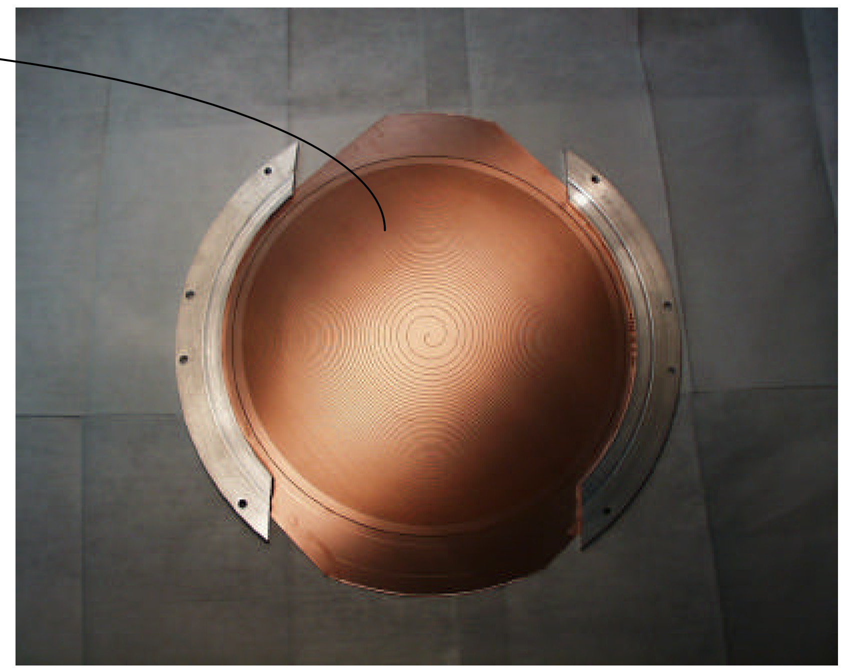
What does a NSE Spectrometer Look Like?
IN11 at ILL was the First



Field-Integral Inhomogeneities cause τ to vary over the Neutron Beam: They can be Corrected

- Solenoids used as main precession fields have fields that vary as r^2 away from the axis of symmetry because of end effects (div B = 0)

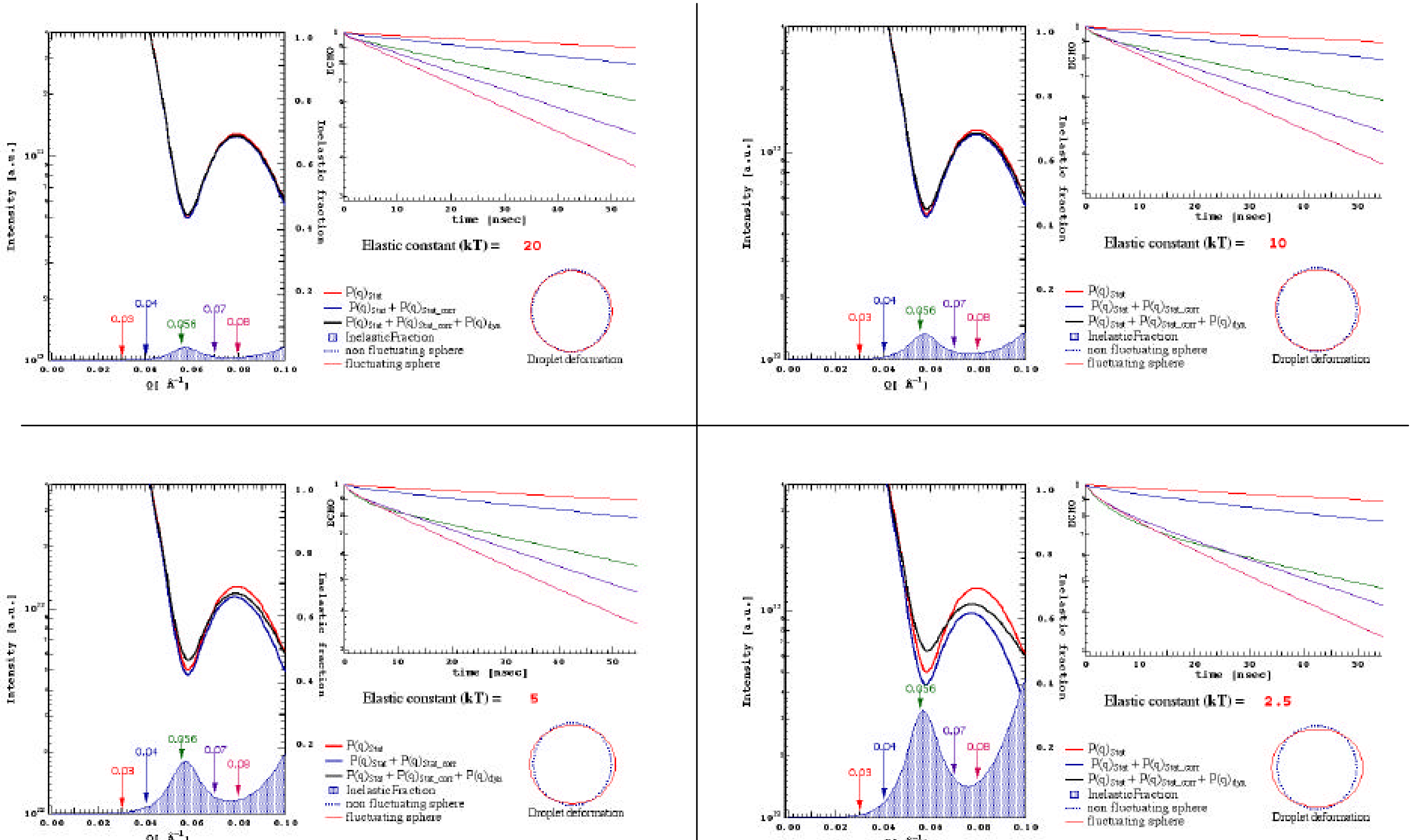




Fresnel correction coil for IN15

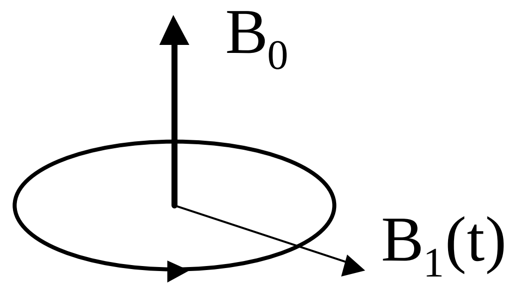
- According to Ampere’s law, a current distribution that varies as r^2 can correct the field-integral inhomogeneities for parallel paths
- Similar devices can be used to correct the integral along divergent paths

Neutron Spin Echo study of Deformations of Spherical Droplets*

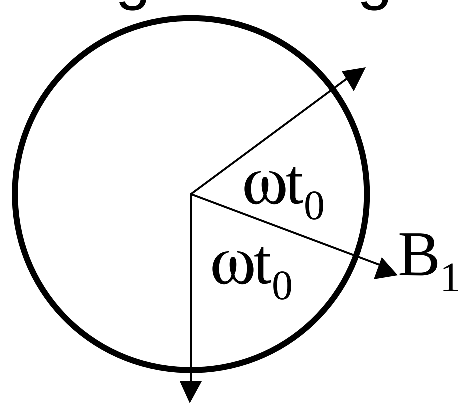


* Courtesy of B. Farago

The Principle of Neutron Resonant Spin Echo



- Within a coil, the neutron is subjected to a steady, strong field, B_0, and a weak rf field $B_1 \cos(\omega t)$ with a frequency $\omega = \omega_0 = \gamma B_0$
- Typically, $B_0 \sim 100$ G and $B_1 \sim 1$ G
- In a frame rotating with frequency ω_0, the neutron spin sees a constant field of magnitude B_1
- The length of the coil region is chosen so that the neutron spin precesses around B_1 thru an angle π.
- The neutron spin angle changes by $2\omega t_0 + \omega d/v$



Neutron Spin Phases in an NRSE Spectrometer*

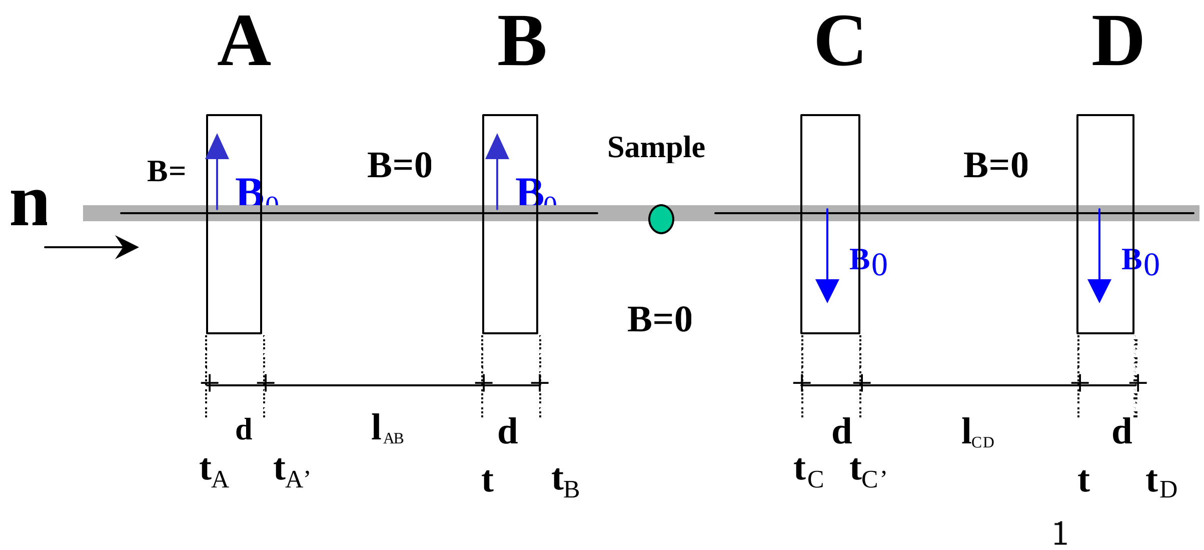


Table 1. Spin orientation

	Time t	Phase field B_r	neutron Spin phase S
A	t_A	ωt_A	0
A'	$t_{A'} = t_A + \frac{d}{v}$	$\omega t_{A'}$	$2\omega t_A + \omega \frac{d}{v}$
B	$t_B = t_A + \frac{l_{AB}+d}{v}$	ωt_B	$2\omega t_A + \omega \frac{d}{v}$
B'	$t_{B'} = t_A + \frac{l_{AB}+2d}{v}$	$\omega t_{B'}$	$2\omega \frac{l_{AB}+d}{v}$
C	t_C	$-\omega t_C$	$2\omega \frac{l_{AB}+d}{v}$
C'	$t_{C'} = t_C + \frac{d}{v}$	$-\omega t_{C'}$	$-\omega \frac{d}{v'} - 2\omega t_C - 2\omega \frac{l_{AB}+d}{v}$
D	$t_D = t_C + \frac{l_{CD}+d}{v'}$	$-\omega t_D$	$-\omega \frac{d}{v'} - 2\omega t_C - 2\omega \frac{l_{AB}+d}{v}$
D'	$t_{D'} = t_C + \frac{l_{CD}+2d}{v'}$	$-\omega t_{D'}$	$2\omega(\frac{l_{AB}+d}{v} - \frac{l_{CD}+d}{v'})$

Echo occurs for elastic scattering when

$$l_{AB} + d = l_{CD} + d$$

* Courtesy of S. Longeville

The Measured Polarization for NRSE is given by an Expression Similar to that for Classical NSE

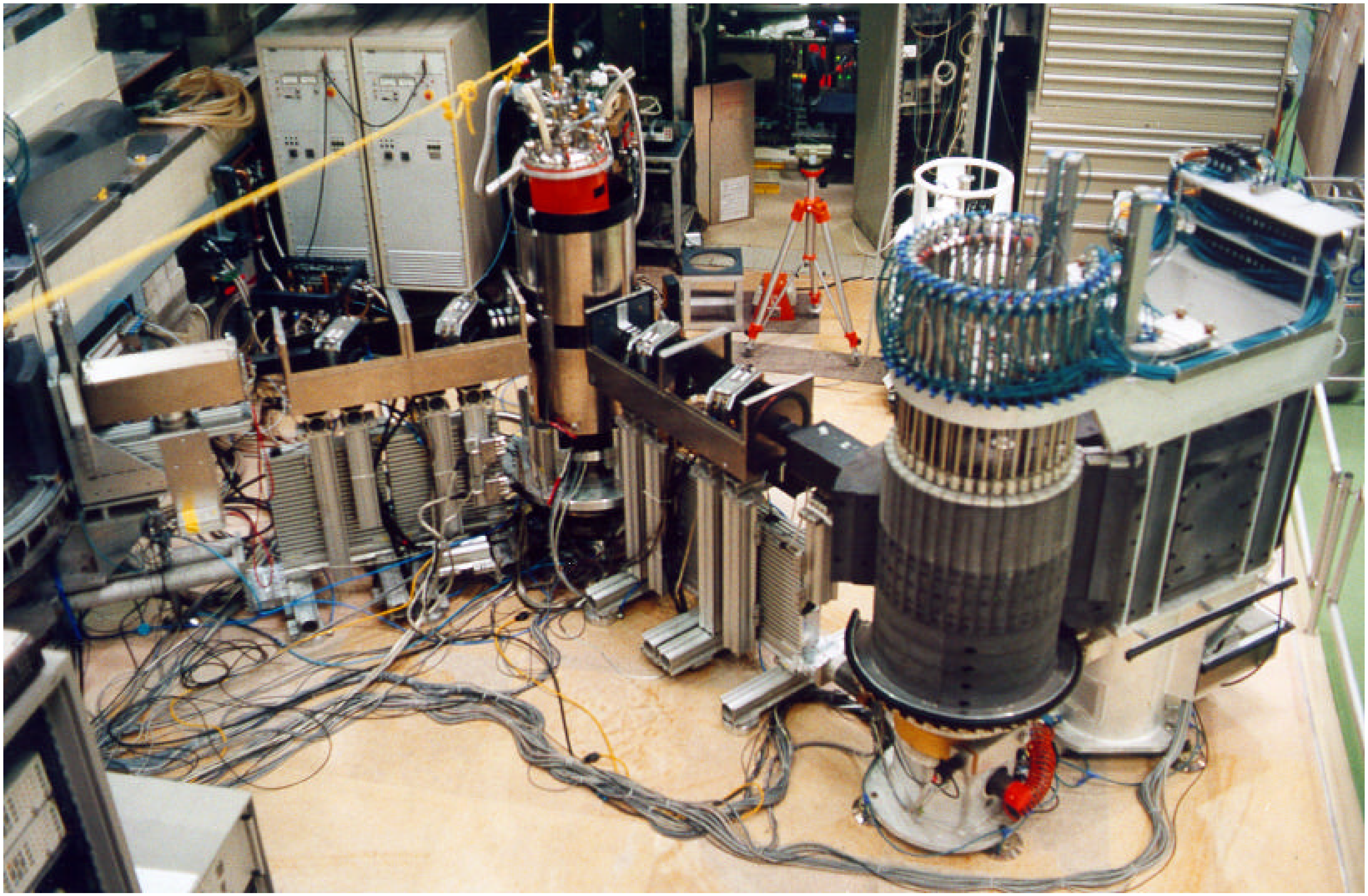
- Again, we assume that *v'* = *v* + δv with δv small and expand to lowest order, giving:

$$\langle P \rangle = \frac{\iint I(\lambda) S(\vec{Q}, \omega) \cos(\omega t_{NRSE}) \, d\lambda \, d\omega}{\iint I(\lambda) S(\vec{Q}, \omega) \, d\lambda \, d\omega}$$

$$\text{where the "spin echo time" } t_{NRSE} = 2\gamma B_0 (l + d) \frac{m^2}{2\pi h^2} \lambda^3$$

- Note the additional factor of 2 in the echo time compared with classical NSE (a factor of 4 is obtained with "bootstrap" rf coils)
- The echo is obtained by varying the distance, l, between rf coils
- In NRSE, we measure neutron velocity using fixed "clocks" (the rf coils) whereas in NSE each neutron "carries its own clock" whose (Larmor) rate is set by the local magnetic field

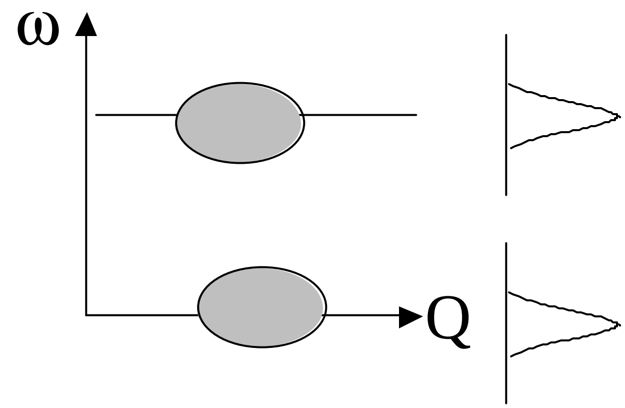
An NRSE Triple Axis Spectrometer at HMI: Note the Tilted Coils



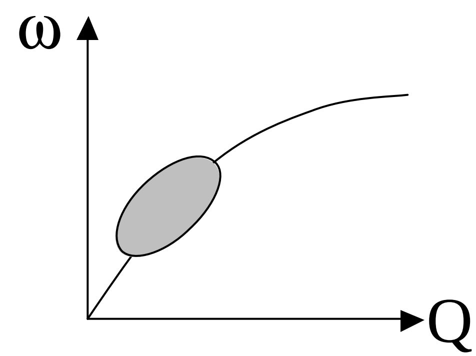
Measuring Line Shapes for Inelastic Scattering

- Spin echo polarization is the FT of scattering within the spectrometer transmission function
- An echo is obtained when

$$\frac{d}{dk_1}\left[\frac{(Bd)_1}{k_1}-\frac{(Bd)_2}{k_2}\right]=0 \;\;\Rightarrow\;\; \frac{N_1}{k_1^2}=\frac{N_2}{k_2^2}$$



- Normally the lines of constant spin-echo phase have no gradient in Q,ω space because the phase depends only on $|k|$
- The phase lines can be tilted by using “tilted” precession magnets



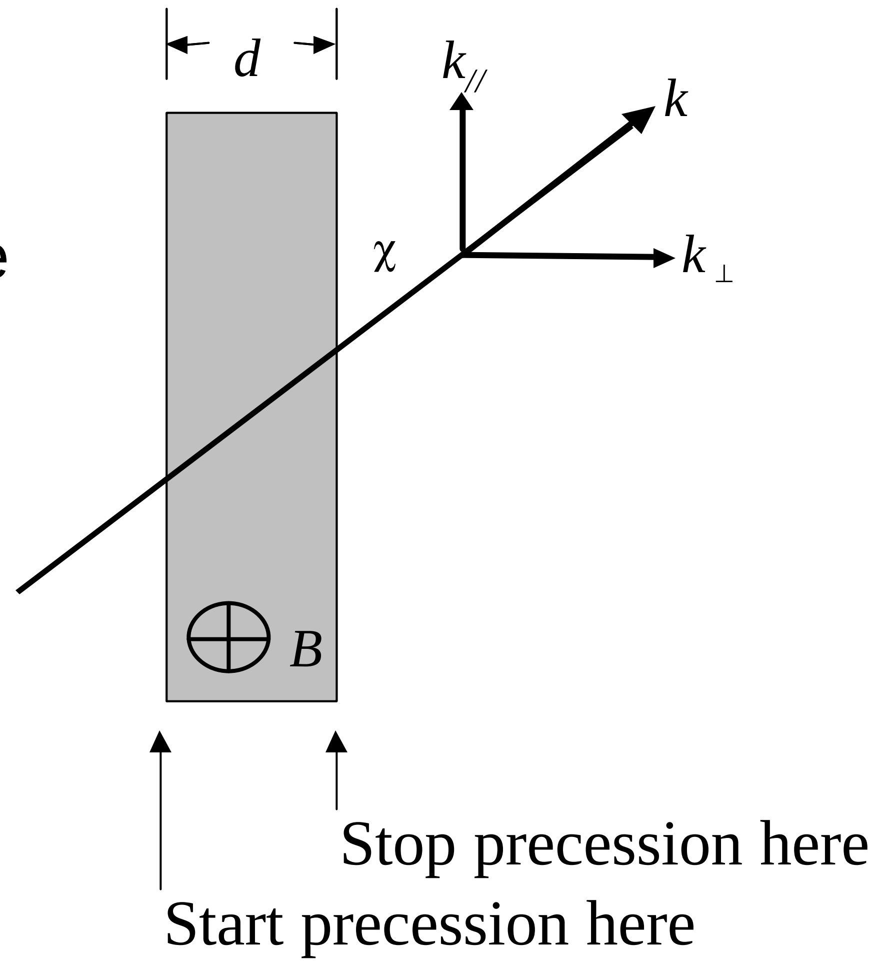
By “Tilting” the Precession-Field Region, Spin Precession Can Be Used to Code a Specific Component of the Neutron Wavevector

If a neutron passes through a rectangular field region at an angle, its total precession phase will depend only on $k_\perp$.

$$w_L = gB$$

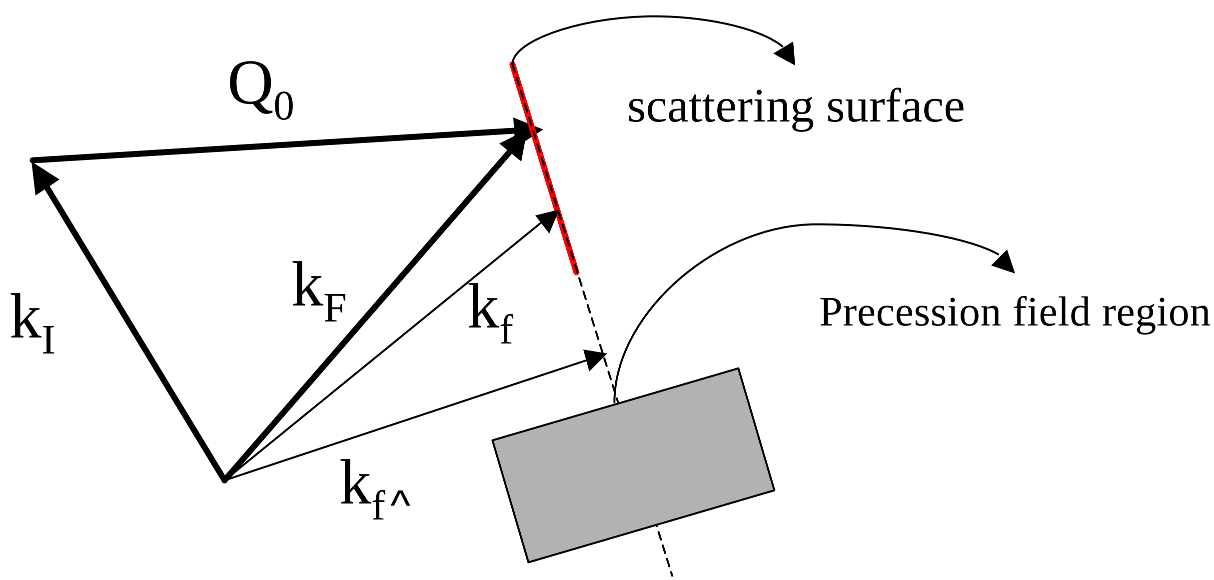
$$f = w_L t = gB \frac{d}{\mathrm{v} \sin c} = \frac{KBd}{k_\perp}$$

with K = 0.291 (Gauss.cm.Å)$^{-1}$



“Phonon Focusing”

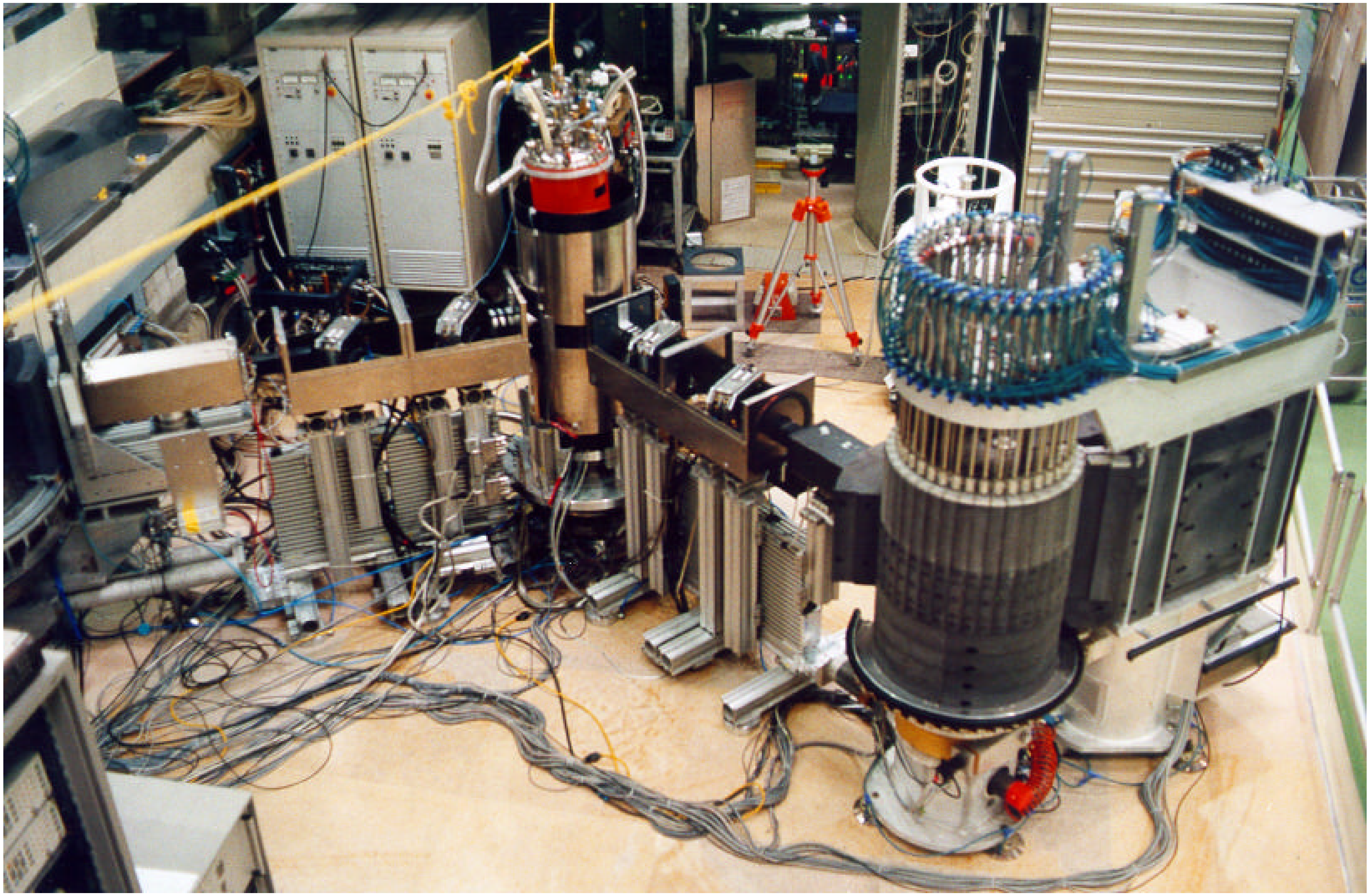
- For a single incident neutron wavevector, $\boldsymbol{k}_I$, neutrons are scattered to $\boldsymbol{k}_F$ by a phonon of frequency ω_o and to $\boldsymbol{k}_f$ by neighboring phonons lying on the “scattering surface”.
- The topology of the scattering surface is related to that of the phonon dispersion surface and it is locally flat
- Provided the edges of the NSE precession field region are parallel to the scattering surface, all neutrons with scattering wavevectors on the scattering surface will have equal spin-echo phase



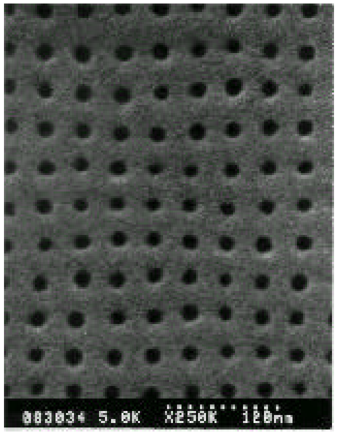
“Tilted Fields”

- Phonon focusing using tilted fields is available at ILL and in Japan (JAERI)….however,
- The technique is more easily implemented using the NRSE method and is installed as an option on a 3-axis spectrometers at HMI and at Munich
- Tilted fields can also be used can also be used for elastic scattering and may be used in future to:
  - Increase the length scale accessible to SANS
  - Separate diffuse scattering from specular scattering in reflectometry
  - Measure in-plane order in thin films
  - Improve Q resolution for diffraction

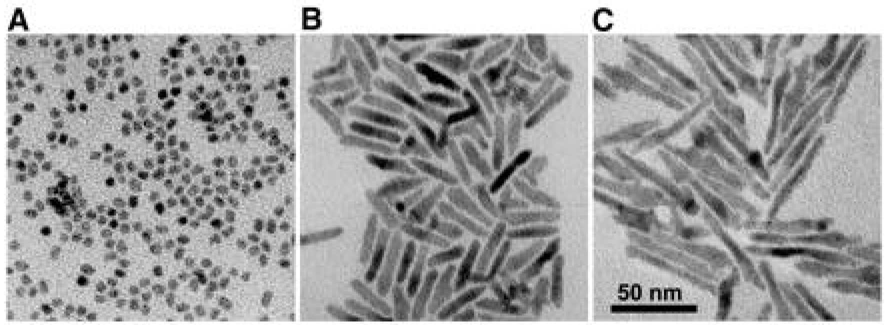
An NRSE Triple Axis Spectrometer at HMI: Note the Tilted Coils



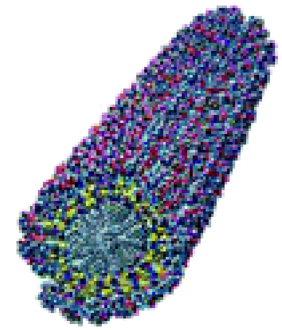
Nanoscience & Biology Need Structural Probes for 1-1000 nm



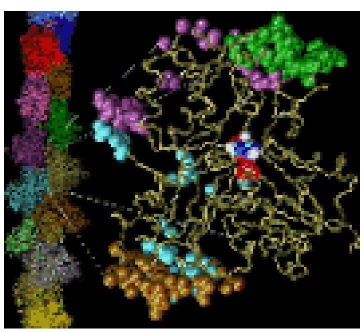
10 nm holes in PMMA



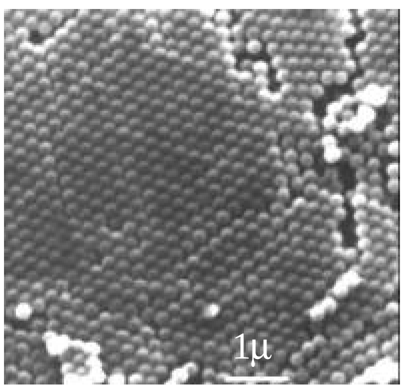
CdSe nanoparticles



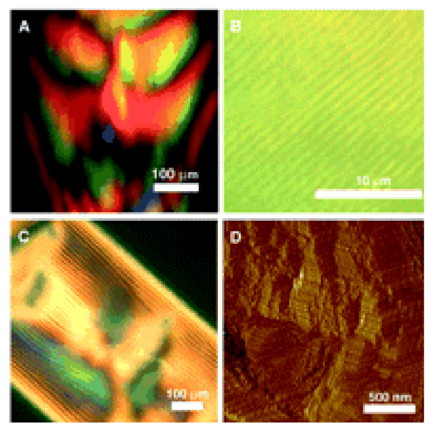
Peptide-amphiphile nanofiber



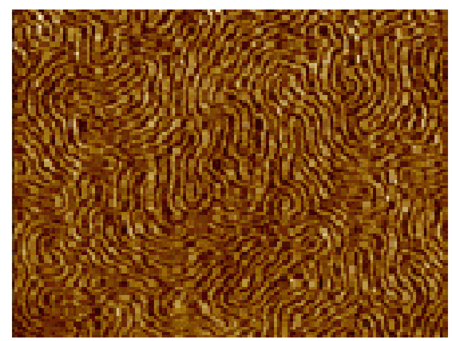
Actin



Si colloidal crystal

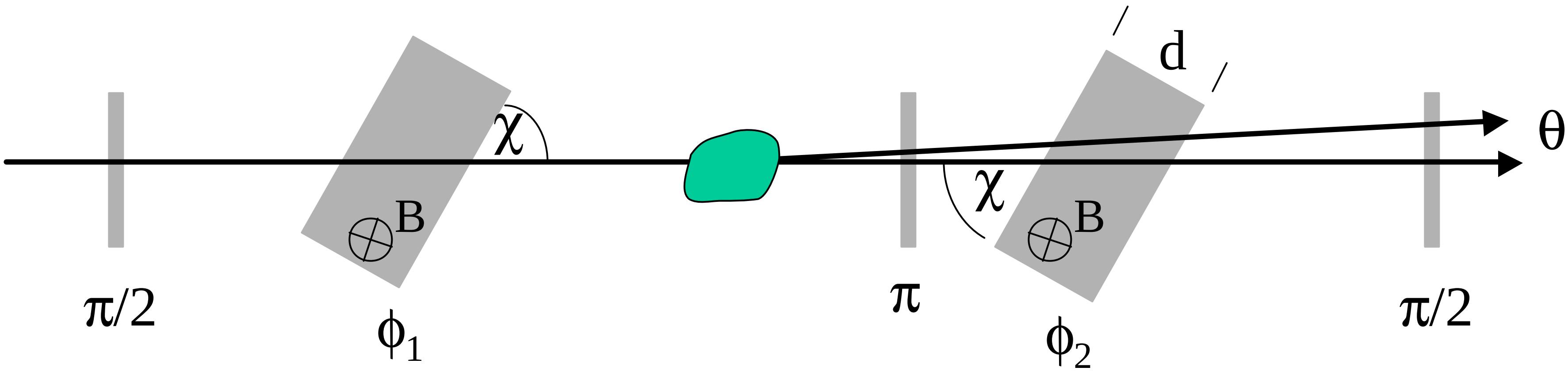


Structures over many length scales in self-assembly of ZnS and cloned viruses



Thin copolymer films

“Tilted” Fields for Diffraction: SANS



- Any unscattered neutron (θ=0) experiences the same precession angles (ϕ_1 and ϕ_2) before and after scattering, whatever its angle of incidence
- Precession angles are different for scattered neutrons

$$f_1 = \frac{KBd}{k \sin c} \text{ and } f_2 = \frac{KBd}{k \sin(c + q)} \Rightarrow \cos(f_1 - f_2) \approx \cos\left[\frac{KBd \cos c}{k \sin^2 c} q\right]$$

$$P = \int dQ.S(Q).\cos\left[\frac{KBd \cos c}{k^2 \sin^2 c} Q\right]$$

Polarization proportional to Fourier Transform of *S(Q)*

Spin Echo Length, $r = KBd \cos c \,/(k \sin c)^2$

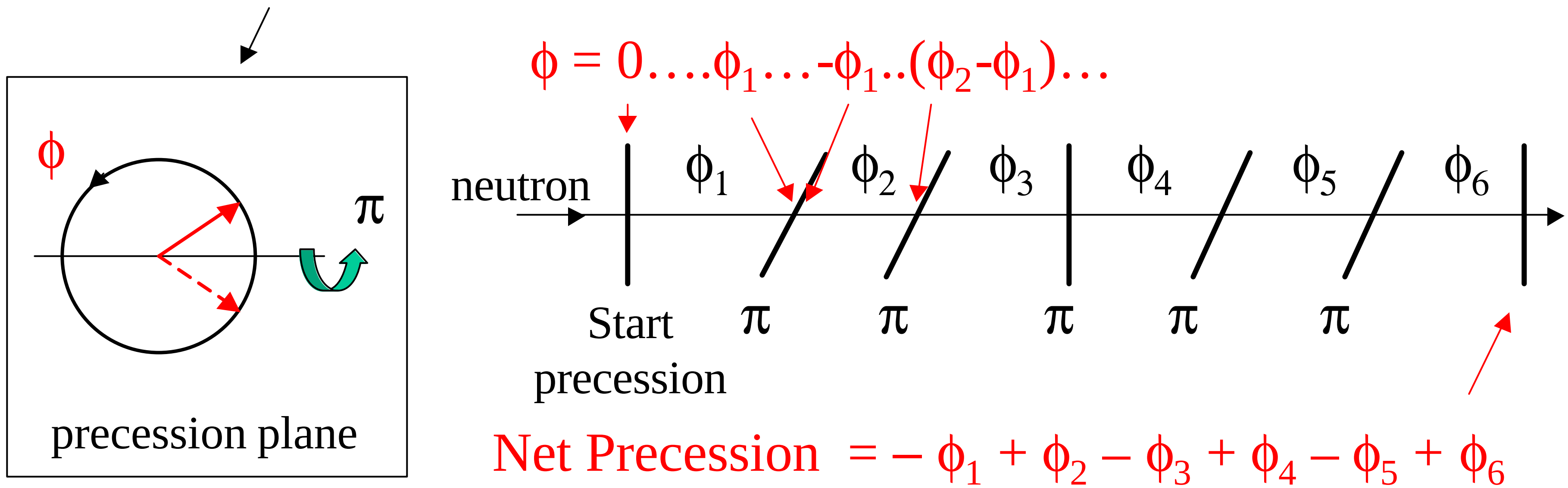
How Large is the Spin Echo Length for SANS?

$Bd/\sin\chi$ (Gauss.cm)	λ (Angstroms)	χ (degrees)	r (Angstroms)
3,000	4	20	1,000
5,000	4	20	1,500
5,000	6	20	3,500
5,000	6	10	7,500

It is relatively straightforward to probe length scales of ~ 1 micron

Using "Sign Reversal" to Implement Angle Coding

An element that performs a π rotation about an axis in the precession plane changes the sign of prior precession angles



- Total net precession angle = $(\phi_4 + \phi_5 + \phi_6) - (\phi_1 + \phi_2 + \phi_3) + 2(\phi_2 - \phi_5)$
- The first two terms depend on neutron velocity only, last term depends on velocity *and* angle of neutron trajectory
  - Requires suitably oriented planar π rotators (flippers)
- Can be set up to encode two, mutually perpendicular, trajectory angles

Conclusion:
NSE Provides a Way to Separate Resolution from Monochromatization & Collimation

- The method currently provides the best energy resolution for inelastic neutron scattering (~ neV)
  - Both classical NSE and NRSE achieve similar energy resolution
  - NRSE is more easily adapted to “phonon focusing”

- The method is likely to be used in future to improve (Q) resolution for elastic scattering
  - Extend size range for SANS (“rescue” scattering from the beam-stop region)
  - May allow 100x gain in measurement speed for SANS
  - Separate specular and diffuse scattering in reflectometry
  - Measure in-plane ordering in thin films (Felcher)