Neutron Optics and Polarization

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With assistance from notes of R. Gähler; ILL Grenoble

1. Neutron waves
2. Neutron guides
3. Supermirrors

   break

4. Neutron polarization
5. Neutron polarimetry
6. Neutron spin transport/flipping

General references:

V.F. Sears, Neutron Optics, Oxford 1989
Rauch and Werner, Neutron Interferometry, Oxford 2000
Fermi: Nuclear Physics (notes by Orear et al. U. Chicago Press - 1949)
QM Text (e.g. Griffiths)

Optics

Optics: the behavior of light (waves) interacting with matter

Waves characterized by wavelength λ

Matter characterized by permeability κ, susceptibility μ, dissipation (ρ/σ)

Interaction characterized by n (index of refraction); δ (skin depth)

Useful when $\lambda >> a$ (atomic spacing)

deBroglie: massive particles behave as waves

$$k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$$

$$k_B T = \frac{p^2 c^2}{2mc^2} \qquad \lambda^2 = \frac{4\pi^2 (\hbar c)^2}{2mc^2 k_B T}$$

$$mc^2 = = 939.6 \text{ MeV}$$

$$\hbar c = 197.3 \text{ MeV-fm} = 1973 \text{ eV-Å}$$

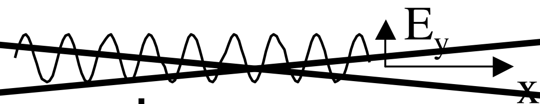
$$v = 2200 \text{ m/s} \qquad \lambda = 1.8 \text{ Å}$$

(thermal neutrons - 300° K)

Note also: $\lambda \propto \sqrt{\frac{1}{T}}$

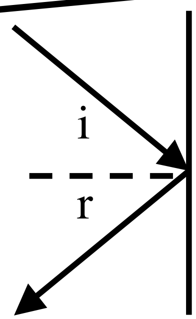
Wave Properties

- Polarization



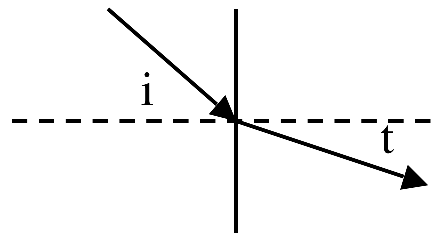
different meaning

- Reflection



(angle of incidence = angle of reflection)

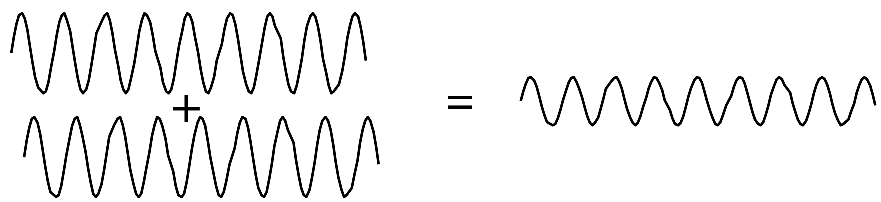
- Refraction



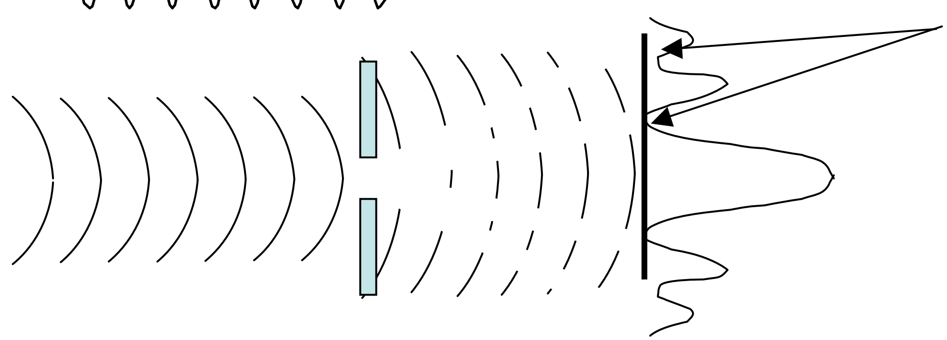
$n_i \sin i = n_t \sin t$

$n = c_0/c \quad c_0 = 2.97 \times 10^8$ m/s

- Interference
  Superposition



- Diffraction



$m\lambda = W \sin\theta$

The wave equations for light and matter waves in vacuum:

$$k = \frac{2\pi}{\lambda} = \frac{p}{\hbar}$$

	EM wave equation ($\vec{\mathbf{E}}, \vec{\mathbf{B}}$)	**Schrödinger equation**
Time dependent:	$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0$	$\nabla^2\Psi + 2i\frac{m}{\hbar}\frac{\partial\Psi}{\partial t} = 0$

$$\Psi(\vec{r},t) = a_k e^{i(\vec{k}\cdot\vec{r} - \omega_k t)}$$

Helmholtz equation:

$$\nabla^2\Psi(\vec{r}) + k^2\Psi(\vec{r}) = 0$$

Time independent Schroedinger equation

Dispersion relations:	$k^2 = \frac{E^2}{(\hbar c)^2}$	$k^2 = \frac{2mE}{\hbar^2}$
Phase velocity:	$v_{ph} = \frac{\omega}{k} = c$	$v_{ph} = \frac{\omega}{k}\sqrt{1 + \frac{m^2c^2}{p^2}} \approx \frac{\omega}{k}\frac{c}{v}$
	$(E = \hbar\omega)$	$\left(E = \sqrt{p^2c^2 + m^2c^4}\right)$

Interactions V(r)

$$\nabla^2\Psi(\vec{r}) + \frac{2m}{\hbar^2}\left[E - V(\vec{r})\right]\Psi(r) = 0$$

Time independent Schroedinger equation

$$\Psi(\vec{r}) = e^{ikz} + \frac{f(\theta)}{r}e^{i\vec{k}\cdot\vec{r}}$$

Incoming plane wave
Outgoing spherical wave

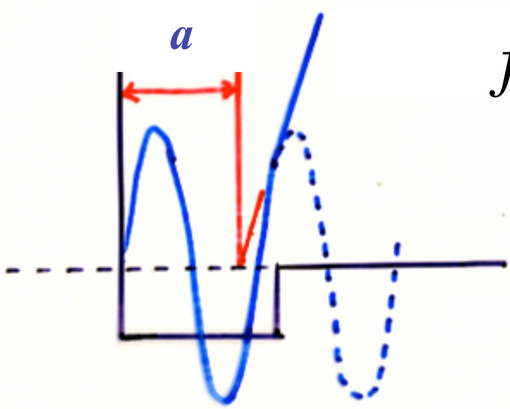
$$f(\theta) = \frac{1}{2ik}\sum_l (2l+1)[e^{2i\delta_l} - 1]P_l(\cos\theta)$$

Partial waves

$$f(\theta) = \frac{1}{2ik}[e^{2i\delta_0} - 1] = \frac{1}{[k\cot\delta_0 - ik]}$$

s-wave scattering

$\delta_0 = -kr_o$ V(r)=0 at r_o



$$f(\theta) = -a + ika^2 + O(k^2)$$

$a = -\frac{\delta_0}{k}$: **scattering length**

(-5 fm < a < 15 fm)

$ka = -\delta_0 \sim 10^{-4}$

$$f(\theta) \approx -a$$

Coherent Scattering Lengths

$$b = \frac{A+1}{A}\,a$$

element	b (fm)
H	-3.74
Be	7.79
C	6.65
Al	3.45
Si	4.15
Ti	-3.44
Fe	9.45
Co	2.49
Ni/^{58}Ni	10.3/14.4
Cu/^{65}Cu	7.72/10.6
Cd	4.87-0.7i

$$f(\theta) = \frac{1}{2ik}[e^{2i\delta_0} - 1] \qquad \delta_0 = -ka$$

Index of refraction

$$\nabla^2\Psi(\vec{r}) + \frac{2m}{\hbar^2}\left[E - V(\vec{r})\right]\Psi(r) = 0$$

Time independent Schroedinger equation

$$\nabla^2\Psi(\vec{r}) + K^2\Psi(\vec{r}) = 0$$

$$K^2 = \frac{2m}{\hbar^2}\left[E - V(\vec{r})\right]$$

For light: $c \Rightarrow \frac{c}{n} = \frac{K}{k}$

$$n(\vec{r}) = \sqrt{1 - \frac{V(\vec{r})}{E}}$$

In general, n is a tensor, i.e. V(r) depends on propogation direction

$$V(\vec{r}) = \sum_i \frac{2\pi\hbar^2}{m} b\,\delta^3(\vec{r} - \vec{r}_i)$$

Fermi Pseudopotenital

(n can also be <1(attractive) and *ii*) complex (absorption or incoherent scattering)

Index of refraction

$$n(\vec{r}) = \sqrt{1 - \frac{V(\vec{r})}{E}}$$

$$V_{nuc}(\vec{r}) = \sum_i \frac{2\pi\hbar^2}{m} b\delta^3(\vec{r} - \vec{r}_i) \approx \frac{2\pi\hbar^2}{m} bN$$

atom number density

$$V_{mag}(\vec{r}) = -\vec{\mu} \cdot \vec{B}_{eff}$$

Materials with Fe, Ni, Co have B_{eff} s.t. $V_{mag} \sim V_{nuc}$

$$n_{\pm}(\vec{r}) = \sqrt{1 - \frac{V_{nuc}(\vec{r}) \mp \vec{\mu} \cdot \vec{B}_{eff}}{E}}$$

Note: neutron magnetic moment μ is negative, i.e. "spin up" has positive V_{mag}.

(n can also be *i*) >1(attractive) ; *ii*) complex (absorption or incoherent scattering)

Notes

$$n = \sqrt{1 - \frac{\lambda^2 N b}{2\pi}}$$

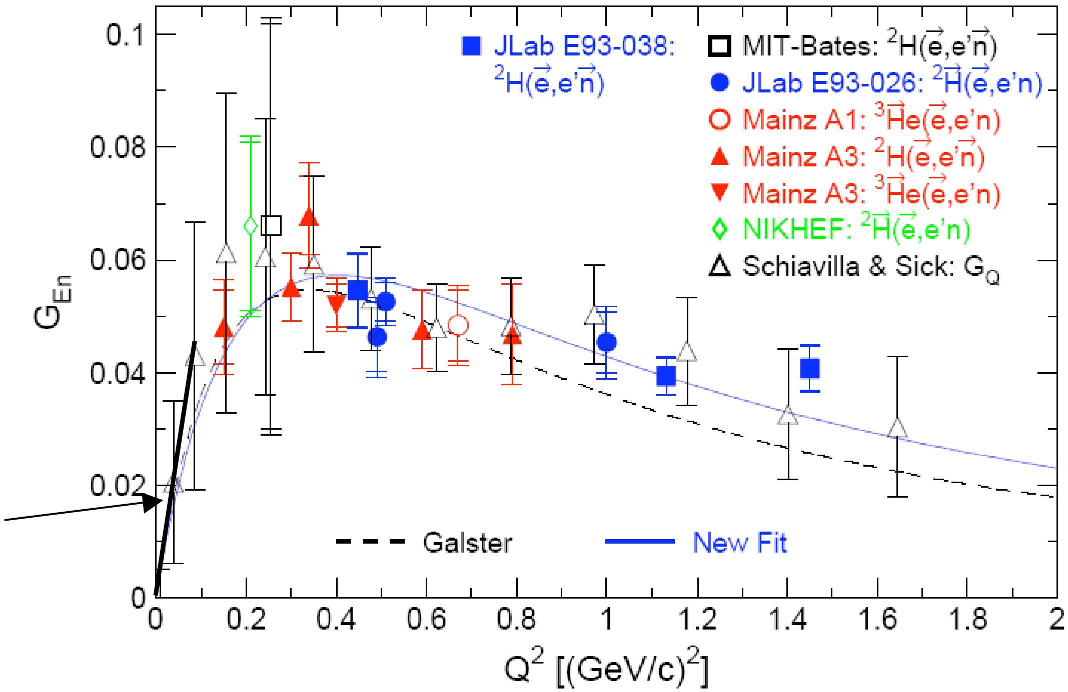
V(r) is generally positive and n<1

Other interactions

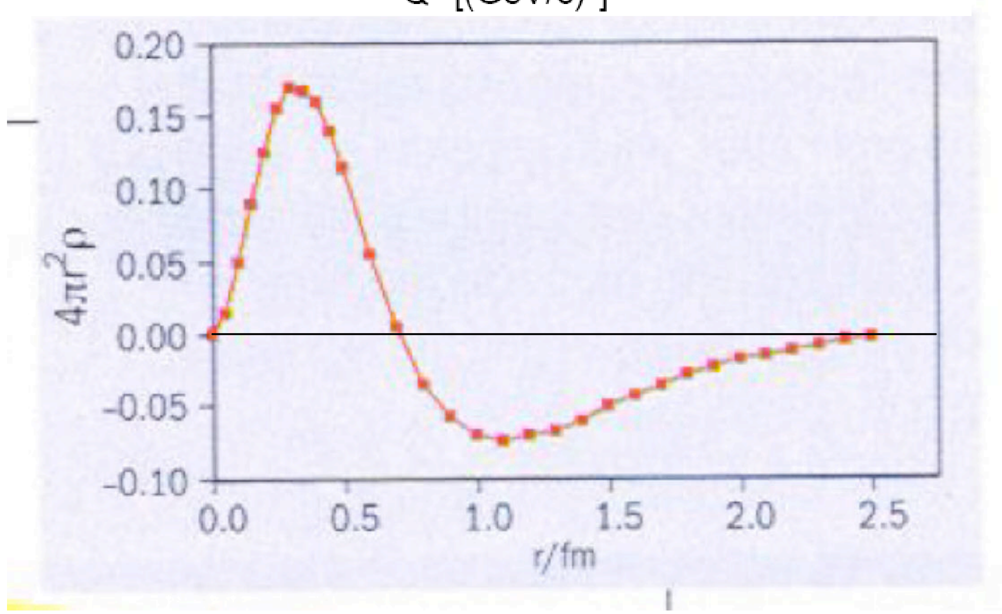
$$V_{spin-orbit}(\vec{r}, \vec{p}) = -\frac{1}{m}\vec{\mu} \times (\vec{p} \times \vec{E})$$

$$V_{electric}(\vec{r}) = \hbar c \alpha \sum_i \int \frac{\rho(r')}{|r_i - r'|} d^3 r$$

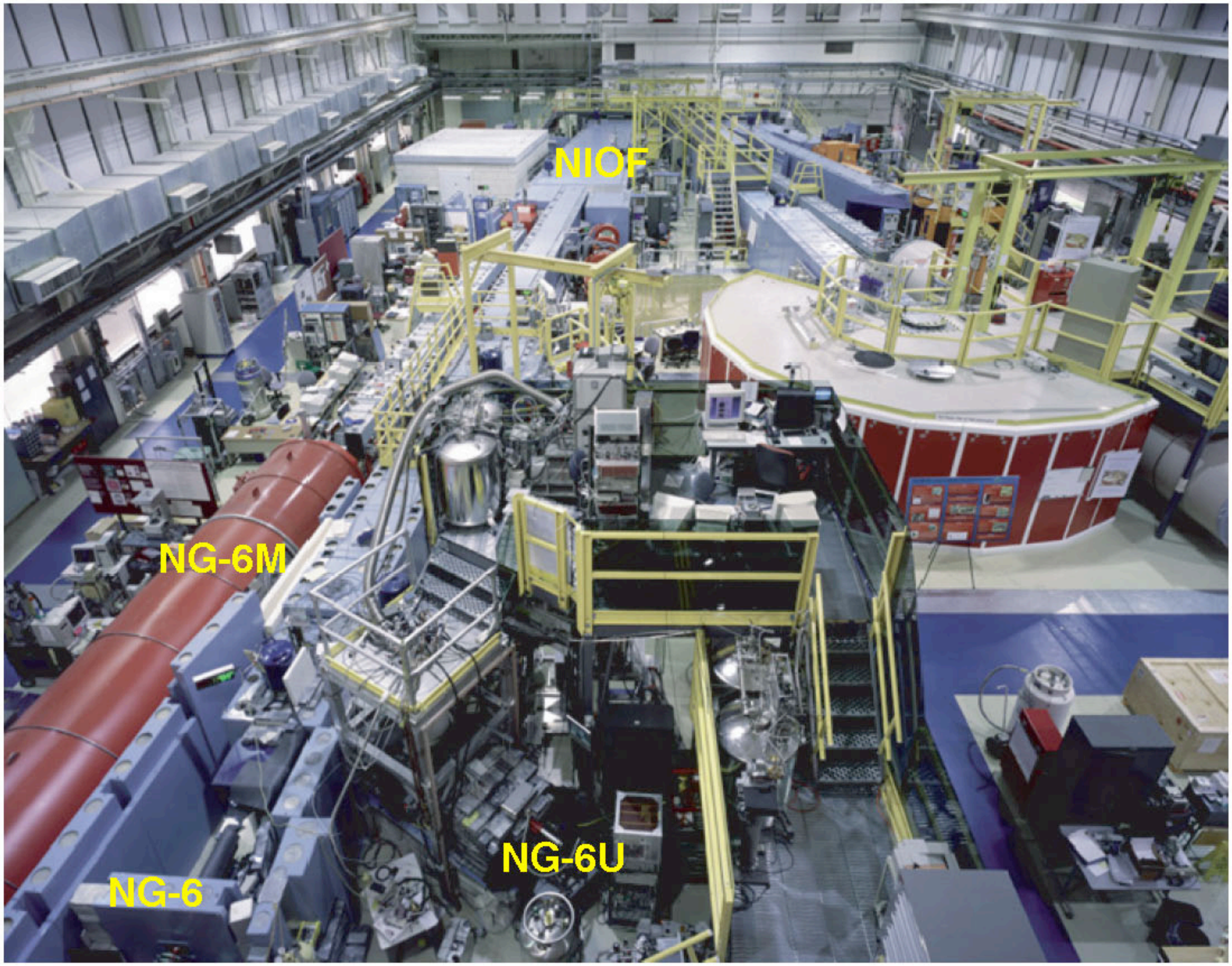
Neutron Charge Distribution



Slope gives $\langle r_q^2 \rangle$ (<0)

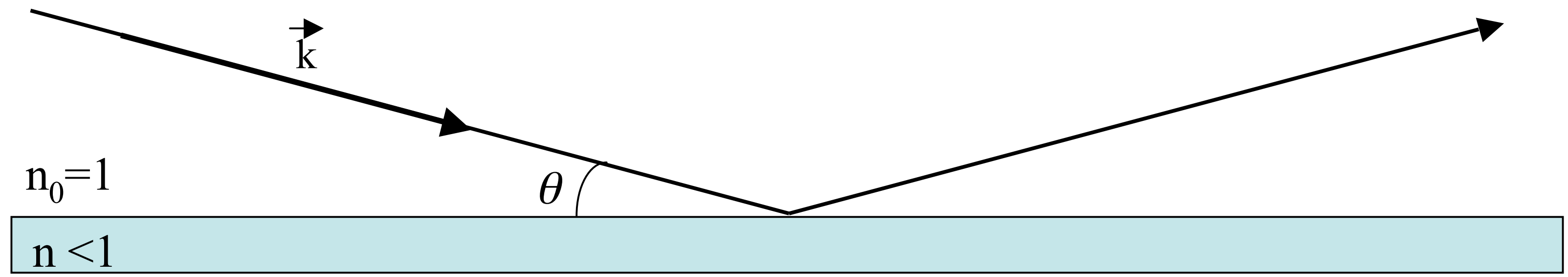


2. Neutron guides



NIST NCNR GUIDE HALL

Neutron guides: **assume no Bragg scattering; absorption negligible;**



$$V = \frac{2\pi\hbar^2}{m} bN$$

N : number density of atoms

b : coherent scattering length

In case of different atoms i, use the weighted average $\langle n_i \cdot b_c \rangle$.

b is generally positive (reflection from edge of square well*) so n<1

*see Peshkin &Ringo, Am. J. Phys. **39**, 324 (1971)

neutrons are totally reflected, if $E_\perp < V$

$$E_\perp = \frac{1}{2} m v_\perp^2 = \frac{\hbar^2 k_\perp^2}{2m} = \frac{2\pi^2\hbar^2}{m\lambda_\perp^2}$$

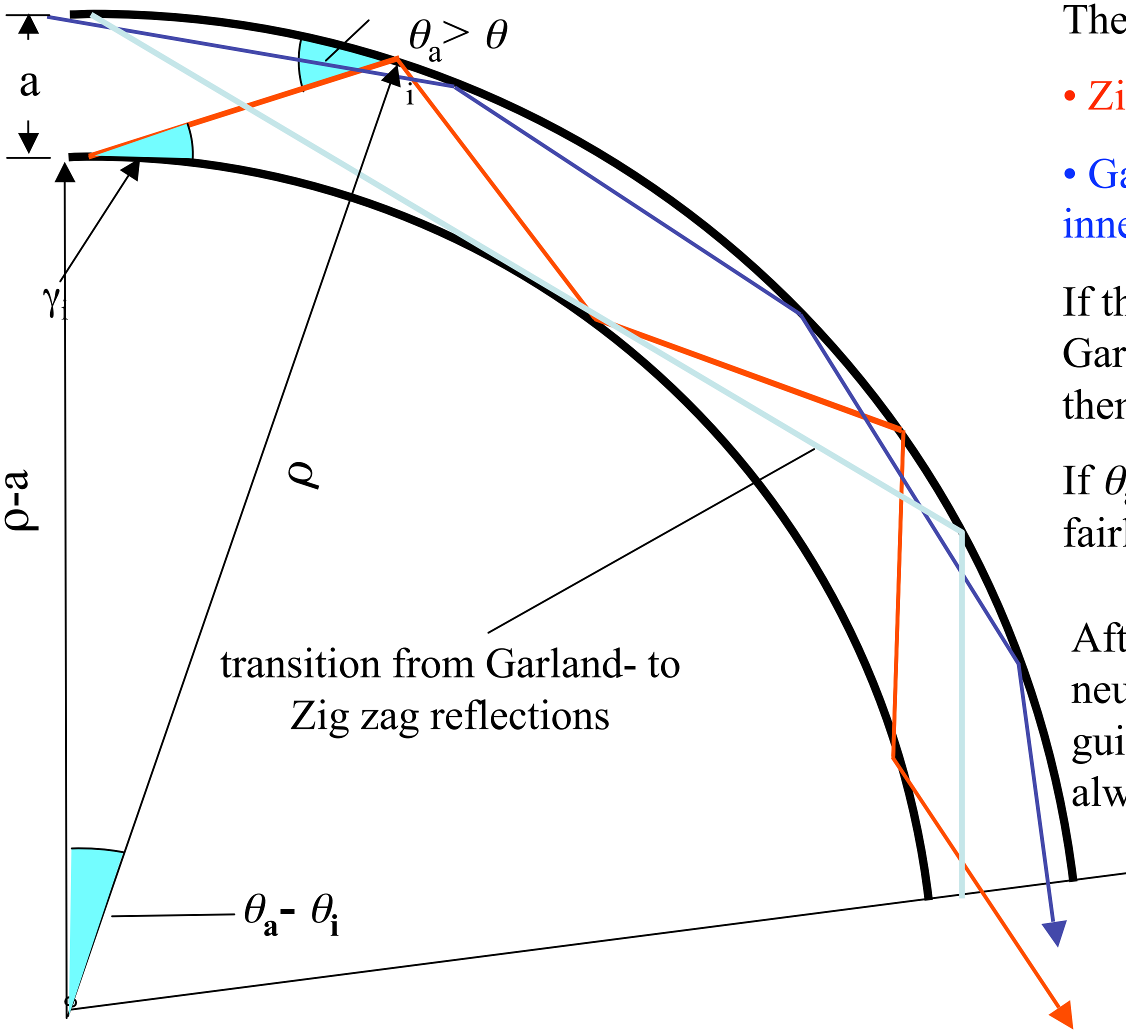
$k_\perp = k\sin\theta$ and $\lambda_\perp = \lambda/\sin\theta$

$$\frac{2\pi^2\hbar^2}{m\lambda_\perp^2} < V \quad \text{or} \quad \sin\theta < \sqrt{\frac{mV}{2\pi^2\hbar^2}}\lambda$$ => critical angle θ_C

For ^{58}Ni, θ_C/λ = 2.03 mrad/Å (1.73 mrad/Å for natural Ni): m=1

Basic properties of ideal bent guides

$$\theta_C = \sin^{-1}\left(\sqrt{\frac{mV}{2\pi^2\hbar^2}}\lambda\right)$$



All refections are assumed to be specular with reflectivity 1 up to a well defined critical angle θ_c and with reflectivity 0 above θ_c .

There are two types of reflections:

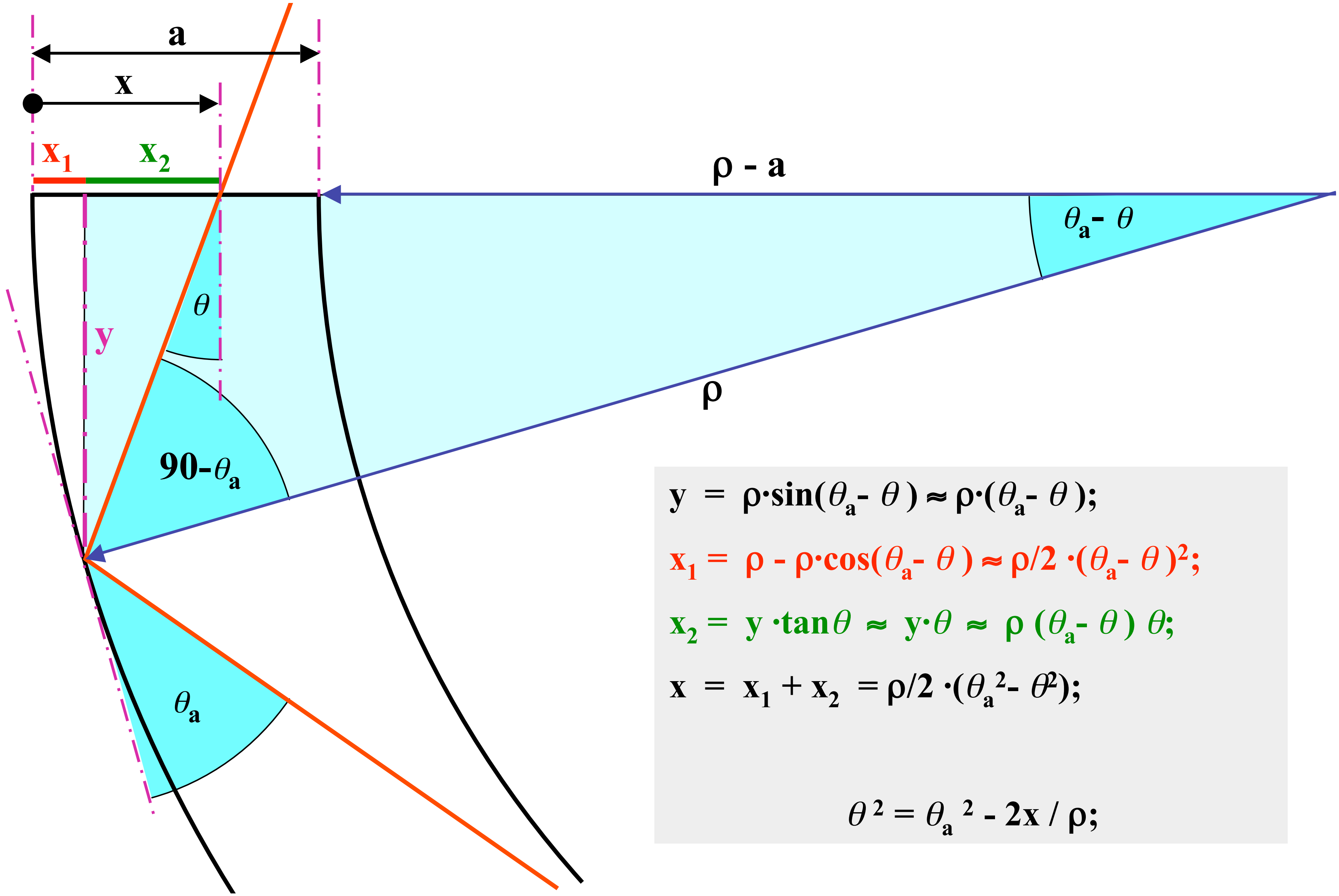
- Zig-zag reflections (large θ_a)
- Garland reflections (never touching the inner wall) (small θ_a)

If the max. reflection angle allows only Garland reflections near the outer wall, then the guide is not efficiently ``filled.''

If $\theta_a \approx \theta_i$ the filling of the guide will be fairly isotropic (many reflections).

After at least one reflection of all neutrons, the angular distribution in the guide is well defined. The angles always repeat.

The Maier-Leibnitz guide formula



$$y \;=\; \rho\cdot\sin(\theta_a - \theta) \approx \rho\cdot(\theta_a - \theta);$$

$$x_1 = \rho - \rho\cdot\cos(\theta_a - \theta) \approx \rho/2\cdot(\theta_a - \theta)^2;$$

$$x_2 = y\cdot\tan\theta \approx y\cdot\theta \approx \rho\,(\theta_a - \theta)\,\theta;$$

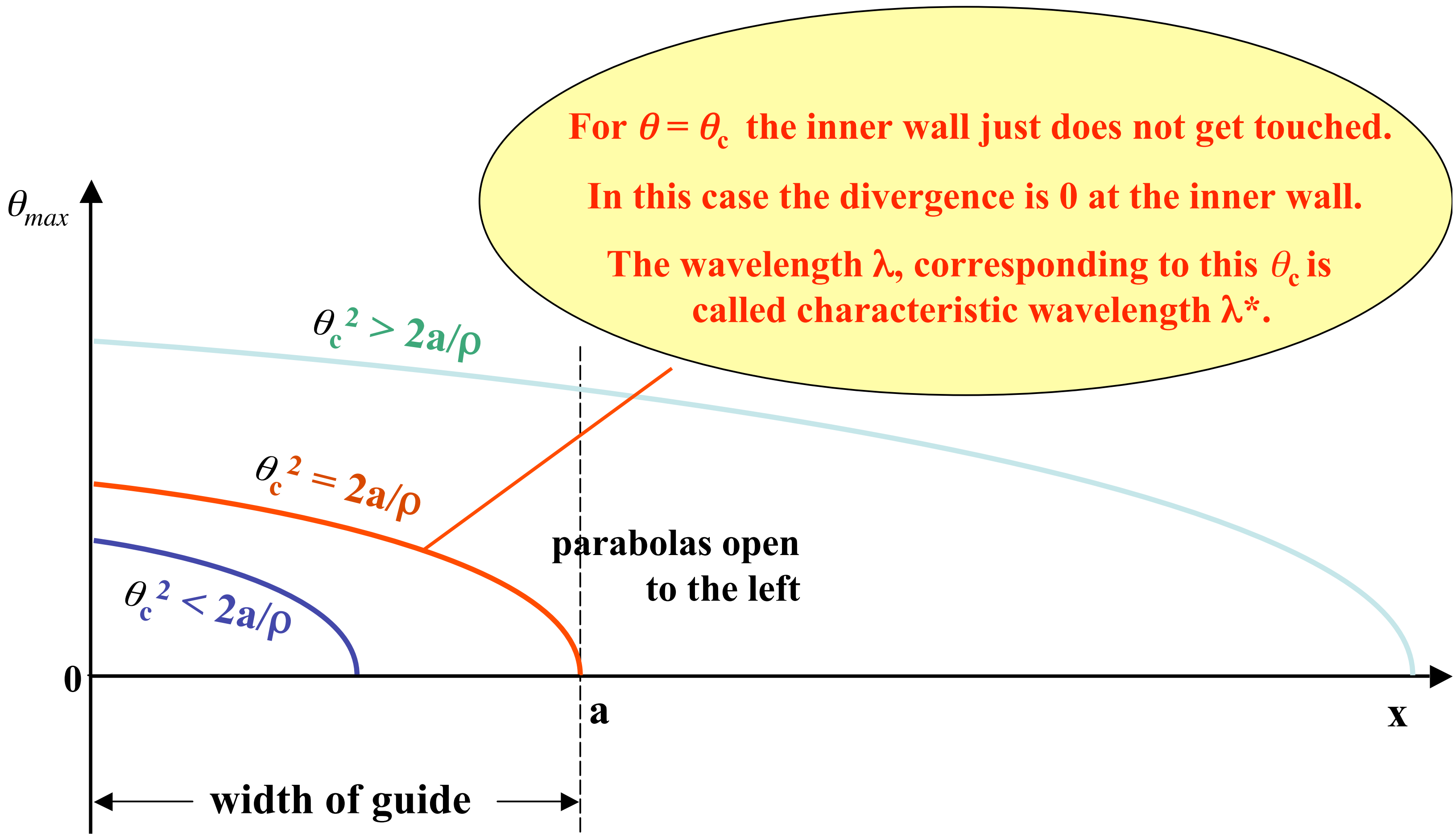
$$x \;=\; x_1 + x_2 = \rho/2\cdot(\theta_a^{\,2} - \theta^2);$$

$$\theta^2 = \theta_a^{\,2} - 2x/\rho;$$

The intensity distribution in a long curved guide is a function of λ

To calculate the max. transmitted divergence, we choose: $\theta_a = \theta_c = k_\perp / k$;

Plot of $\theta_{max} = (\theta_c{}^2 - 2x/\rho)^{1/2}$ **as function of x for different** θ_c **shows added divergence:**



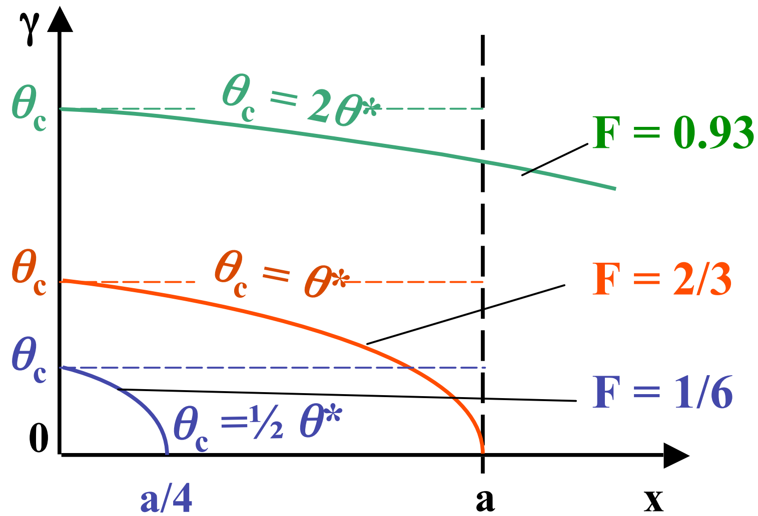
Filling factor F (intensity ratio curved guide/straight guide):

$$\theta_{max}^2 = \theta_a^{\ 2} - 2x / \rho$$

$$F = \frac{1}{a\theta_c}\int_0^{a^*} \theta dx = \frac{1}{a}\int_0^{a^*} \sqrt{1 - \frac{2x}{\rho\theta_c^2}}dx$$

$\theta^* = (2a/\rho)^{1/2}$

$a^* = \gamma_c \rho/2$



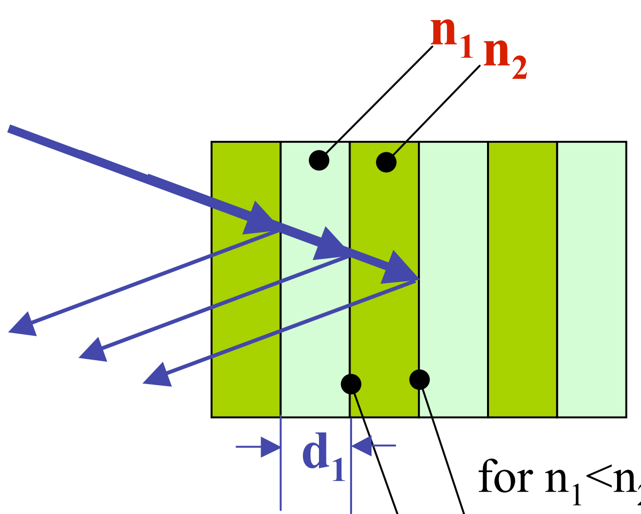
$F(\theta_c = \theta^*) = 2/3$; $F(\theta_c = 2\theta^*) = 0.93$; $F(\theta_c = \theta^*/2) = 1/6$;

Multilayer mirrors and supermirrors:

Can we exceed the critcal angle of "total external reflection?" $\theta_C = \sin^{-1}\left(\sqrt{\frac{mV}{2\pi^2\hbar^2}}\lambda\right)$

Multilayers provide reflections from multiple interfaces between different indexes of refraction

Incident plane wave; λ_0



Reflectivity for normal incidence at one interface

$$\mathbf{R} = \left(\frac{\mathbf{n_1 - n_2}}{\mathbf{n_1 + n_2}}\right)^2$$

for $n_1<n_2$

constructive interference for $\lambda = 2n_1 d_1 \sin\theta_1$

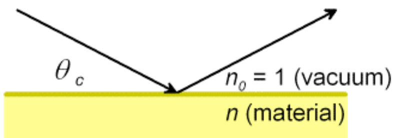
destructive interference for $\lambda = 2n_2 d_2 \sin\theta_2$

Let $n_1 d_1 = n_2 d_2$

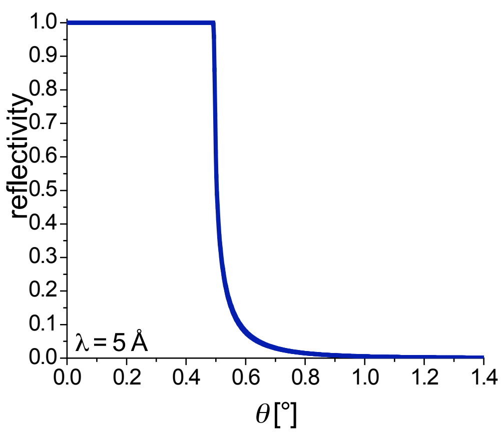
Pick n_1+n_2 small: only a few layers provides high reflectivity for mirror FOR CONSTANT $\lambda \sin\theta_i$

Supermirror: vary nd for quasi-continuous λ and θ_i

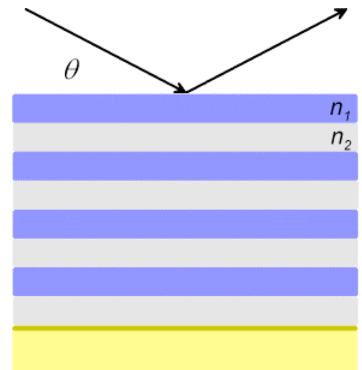
single mirror



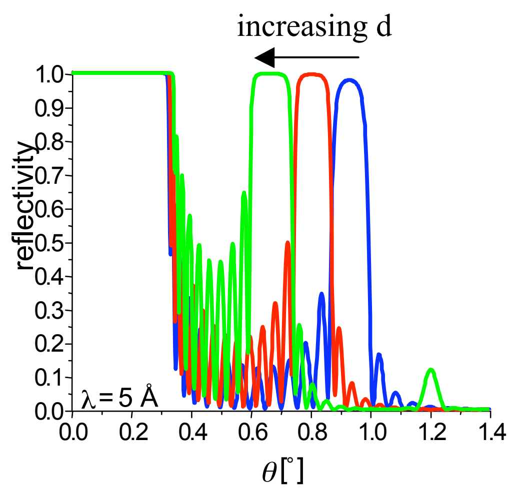
- refractive index n < 1
- total external reflection
  e.g. Ni θ_c = 0.1 °/Å



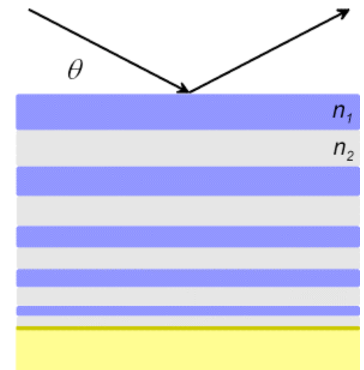
multilayer

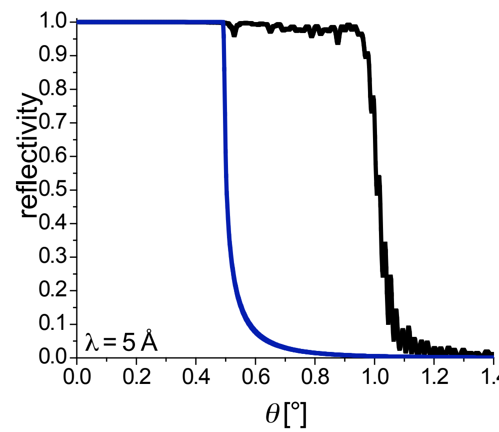


$$\lambda = 2nd\sin\theta$$



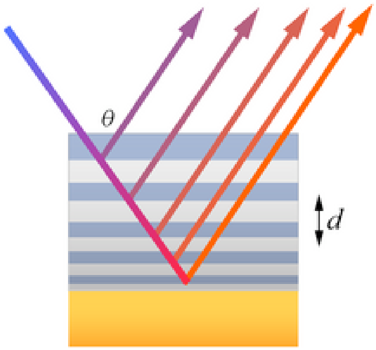
supermirror

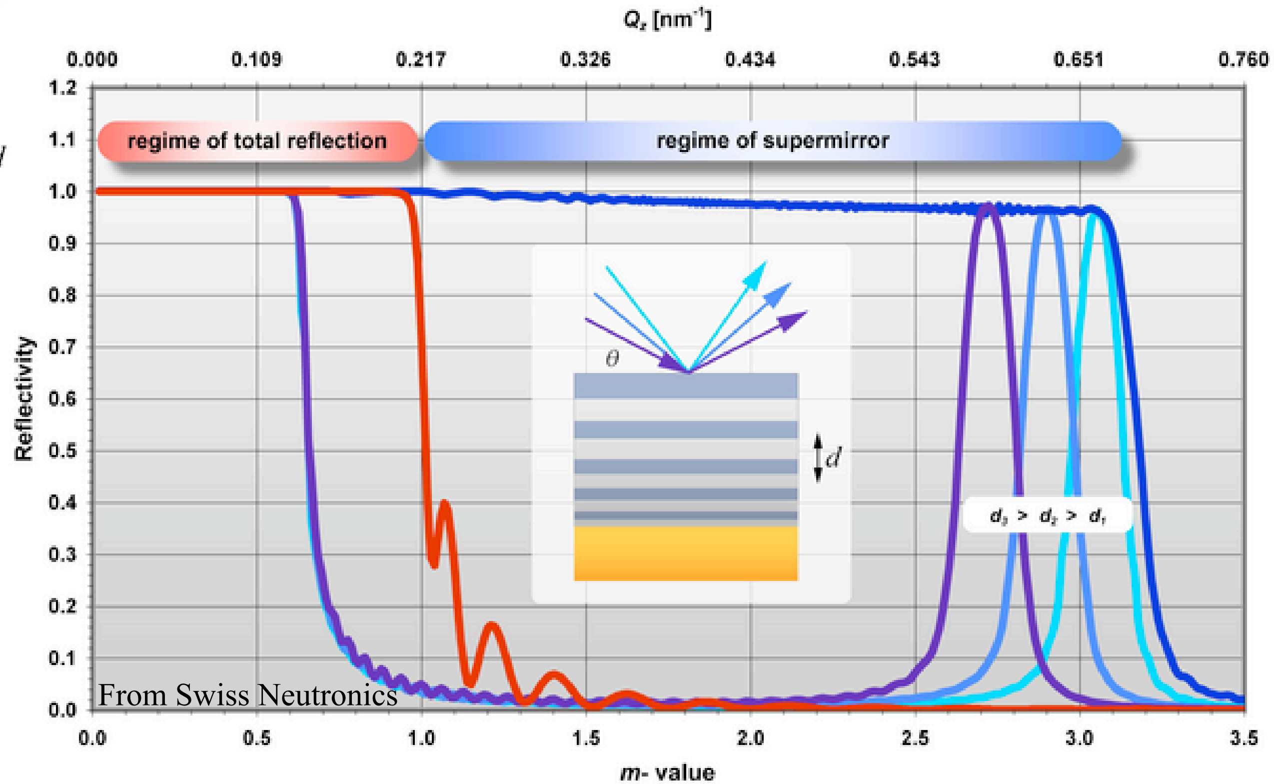




Multilayer supermirrors: Characterized by $m = \frac{\theta_{mirror}}{\theta_{Ni}}$

θ_{Ni} ← 1.73 mrad/Å





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F. Mezei, Novel polarized neutron devices: supermirror and spin component amplifier, Communications on Physics 1, 81, 1976.

F. Mezei, P. A. Dagleish, Corrigendum and first experimental evidence on neutron supermirrors, Communications on Physics 2, 41, 1977.

J. B. Hayter, H. A. Mook, Discrete Thin-Film Multilayer Design for X-Ray and Neutron Supermirrors, J. Appl. Cryst. 22, 35, 19892, 35, 1989.

O. Schaerpf, Physica B 156&157 631, 639 (1989)

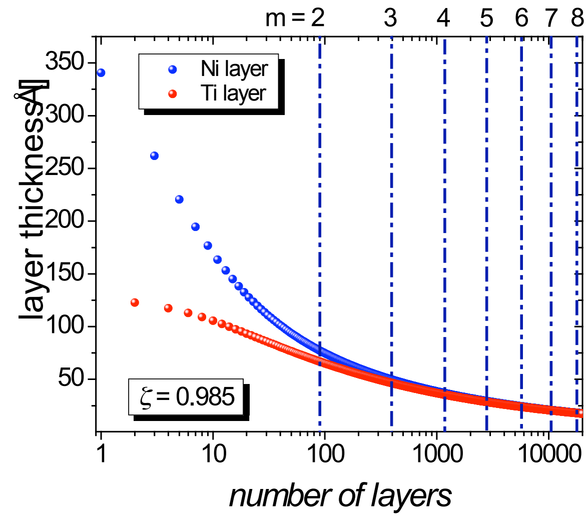
layer sequence λ/4 layer thickness, overlapp of superlattice Bragg peaks

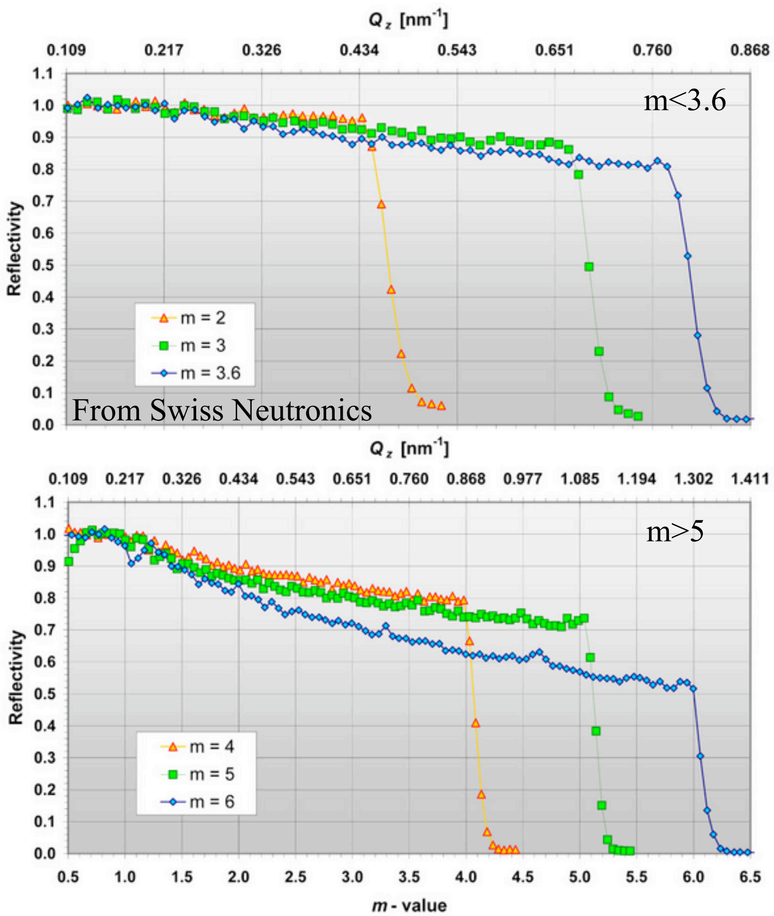
[1] J. B. Hayter, H. A. Mook, *Discrete Thin-Film Multilayer Design for X-Ray and Neutron Supermirrors*, J. Appl. Cryst. **22** (1989) 35

high *m* - value - more angles

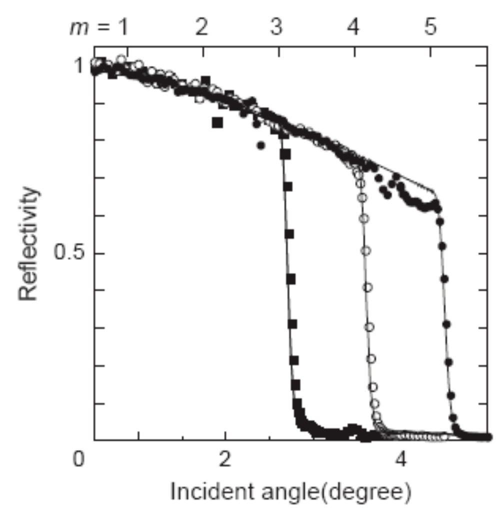
=> large number of layers, e.g.

$m = 2 \Rightarrow$ 120 layers (R ≥ 90%)
$m = 3 \Rightarrow$ 400 layers (R ≥ 80%)
$m = 4 \Rightarrow$ 1200 layers (R ≈ 75%)
$m = 5 \Rightarrow$ 2400 layers (R ≈ 63%)





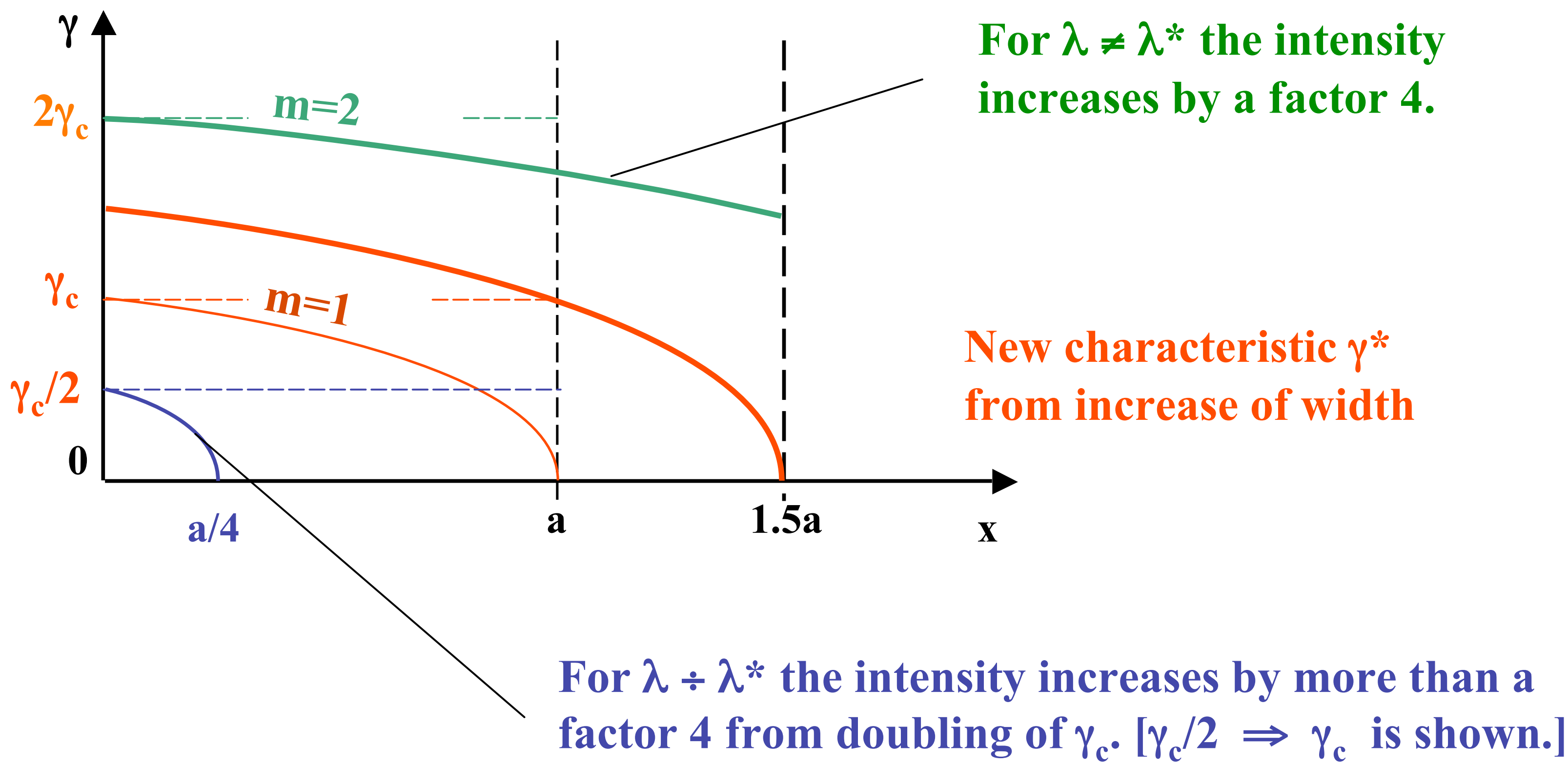
M. Hino et al., NIMA. **529** (2004) 54



m>1 guides

Replacing a Ni guide (m=1) by a SM guide (m=2) doubles $k_\perp$ and γ_c.

Increasing the guide width from a ⇒ 1.5a increases γ* by $1.5^{1/2}$ and also increases the direct line of sight $[L_d = (8a\rho)^{1/2}]$ by $1.5^{1/2}$.



Guide losses

Reflectivity:

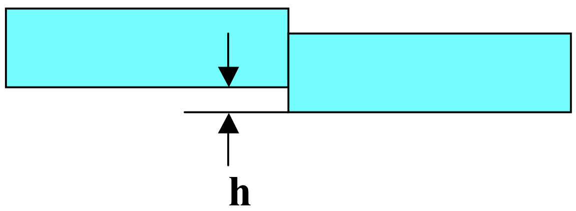
Garland refl.: $l_g = 2\,\rho\,\gamma$; Zig-zag refl.: $l_z = \rho\,(\gamma_a - \gamma_i)$; or $l_z \approx d/\gamma$;

$R^n = (1 - \Delta)^n \approx 1 - n\,\Delta$; for $\Delta << 1$;

for $R = 0.97$ and $L = 1.2\,L_0$; $n \geq 3$ ($\rho = 2700$m; $\gamma = 1/200$)

$\Rightarrow$ beam transmission f_R by reflectivity R: $< f_R > = 0.9$; higher for long SM-guides!

l = mean length for one reflection from side walls;
n = mean number of reflections; R = reflectivity; L_0 = length of free sight; L=100m



Alignment errors: Gauss distrib.: $f(h) = \exp(-h^2/h_f^2) / (\pi^{-1/2}\,h_f)$

Transmission $f_a = 1 - L/L_p \cdot h_f \cdot \pi^{-1/2} / a$;

For $L = 1.2\,L_0$; a = 3 cm, L_p = 1m, L = 100m and $h_f = 20\ \mu$m:

$\Rightarrow$ mean transmission due to alignment errors: $f_a = 0.96$;

For guide of 20x3 cm, the necessary precision on top / bottom is 7 times worse (0.14 mm).
h_f = mean alignment error; a = guide width (30 mm); L_p = length of plates;

Guide losses

waviness:

Let the neutron be reflected under angle $\gamma + \alpha$ instead of γ.

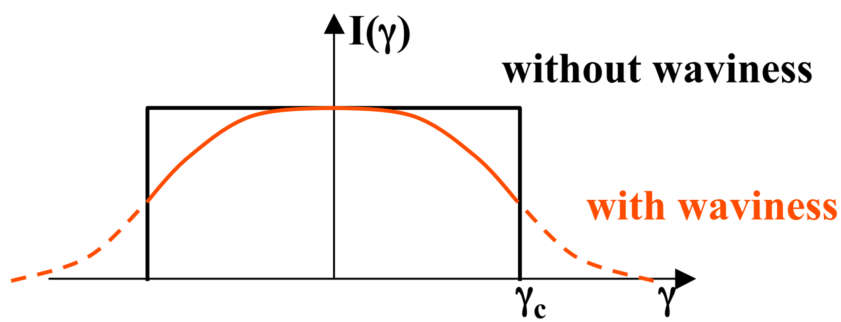
$$g(\alpha) = \frac{1}{\sqrt{\pi} \cdot \alpha_w^2} \exp\left(- \alpha^2 / \alpha_w^2 \right)$$

For 6 reflections, $L = 1.2\ L_0$; $k = 0.7k^*$: $f_w = 1 - \alpha_w / \gamma^*$;

For $\alpha_w = 10^{-4}$, $\gamma^* = 1.7 \cdot 10^{-3}$ [1Å, Ni]: $f_w = 0.94$;

The outer areas of the intensity distribution $I(\gamma)$ are more affected than the inner ones.

f_w : transmission due to waviness; α_w = RMS waviness; γ^* = characteristic angle of guide



$\alpha_w = 2 \cdot 10^{-4}$ for the new guides seems acceptable (m=2!)

The angle $\Delta\alpha$ between the guide sections can be treated as waviness. $\Delta\alpha = 1/27000$ for H2.

Summary of main guide formulas

ρ = radius of curvature
a = width of guide
x = width from outer surface
$k_\perp$ = max. vertical wavevector;
γ = angular width w.r.t. guide axis
$\gamma_c \approx k_\perp / k = (4\pi N b)^{1/2} / k = \lambda (N b/\pi)^{1/2}$

$k_\perp$ [Ni] = $1.07 \cdot 10^{-2}$ Å^{-1} $\leftrightarrow$ m=1;
$\Rightarrow$ $k_\perp = 1.07 \cdot x \cdot 10^{-2}$ Å^{-1}; [SM: m=x]
Supermirrors of n-guides typically have m =2
$k_\perp$ [glass] = $0.63 \cdot 10^{-2}$ Å^{-1};
$k_\perp$ [Ni-58] = $1.27 \cdot 10^{-2}$ Å^{-1};

$\gamma = (\gamma_c^{\,2} - 2x/\rho)^{1/2}$; γ = angular width at output of curved guide; γ_c = critical angle of total reflection;

$\Rightarrow$ **$\gamma^* = (2a/\rho)^{1/2}$;** γ^* = characteristic angle;

conditions: each neutron is at least once reflected at outer surface and reflectivity is step function up to γ_c;

$\lambda^* = 2\pi\,(2a/\rho)^{1/2} / k_\perp$; λ^* = characteristic wavelength;

Flux $d\phi/d\lambda$ for given source brilliance $d^2\phi/d\lambda d\Omega$ and distance z from source:

$$\frac{d\Phi}{d\lambda} = \int_{source} \frac{d^2\Phi}{d\lambda d\Omega}\frac{1}{z^2}dF$$

Flux $d\phi/d\lambda$ in long straight guide for constant source brilliance $d^2\phi/d\lambda d\Omega$
if angular acceptance in guide $\gamma_x\,\gamma_y$ is smaller than angular emittance of source:

$$\frac{d\Phi}{d\lambda} = \frac{d^2\Phi}{d\lambda d\Omega}\gamma_x\gamma_y$$

Summary of main guide formulas

$$F = \frac{1}{a\gamma_c}\int_0^{a^*} dx\,\gamma = \frac{1}{a}\int_0^{a^*} dx\left(1 - \frac{2x}{\rho\cdot\gamma_c^2}\right)^{1/2}$$

$a^* = a$ for $\gamma_c \geq \gamma^*$;
$a^* = \rho\,\gamma_c{}^2/2$ for $\gamma_c \leq \gamma^*$;

F = filling factor of guide

$L_0{}^2 = 8\rho a$; L_0 = direct line of sight of bent guide;

$\Delta = a-(L_0/2-dL)^2/2R$; Δ = width of direct sight of bent guide; dL = missing length to L_0

$x_b = L^2/(2\rho)$; x_b = lateral deviation from start direction; L = length of guide;

$n_g = L/(2\,\rho\,\gamma)$; $n_z = L/(\rho\,(\gamma_a - \gamma_i))$; or $n_z \approx L\,\gamma\,/a$; n = number of reflections; n_g for Garland; n_z for Zig-zag refl.;

$f_w = 1 - \alpha_w\,/\gamma^*$; f_w = loss due to waviness; α_w = RMS value of waviness; L = 1.2 l_d;

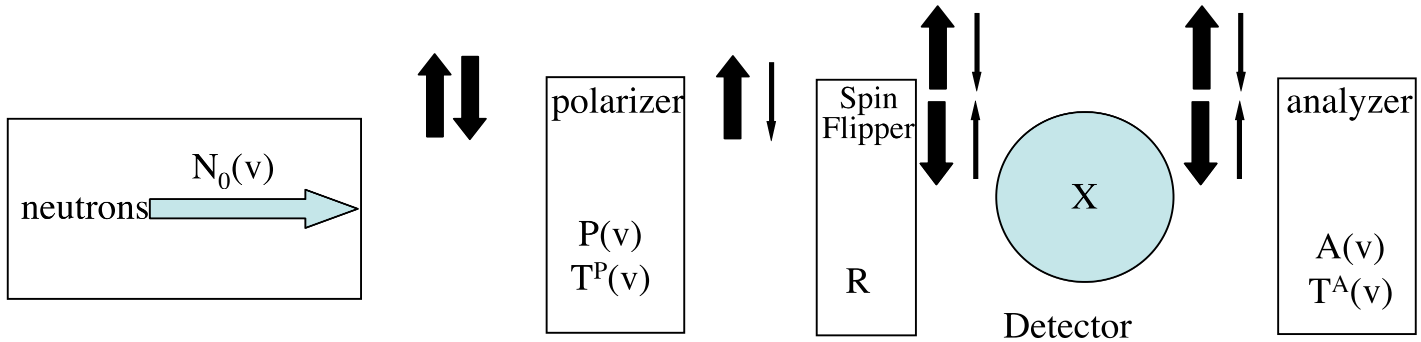
$f_a = 1 - L/L_p\cdot h_f\cdot\pi^{-1/2}\,/a$; f_a = loss due to steps; h_f = RMS value of alignment error;

Loss V in intensity due to gap of length L for a guide of cross scection a×b:

$$V = \frac{L\cdot\gamma_c}{2}\left(\frac{1}{a} - \frac{1}{b}\right) - \frac{L^2\cdot\gamma_c^2}{4ab}$$

Break

Neutron Polarization and Polarimetry



$$R_{Exp} = \Sigma(\uparrow + \downarrow) + \Delta(\uparrow - \downarrow) = N_0 T_1 T_2 T_P\, [\Sigma + \Delta P R]$$

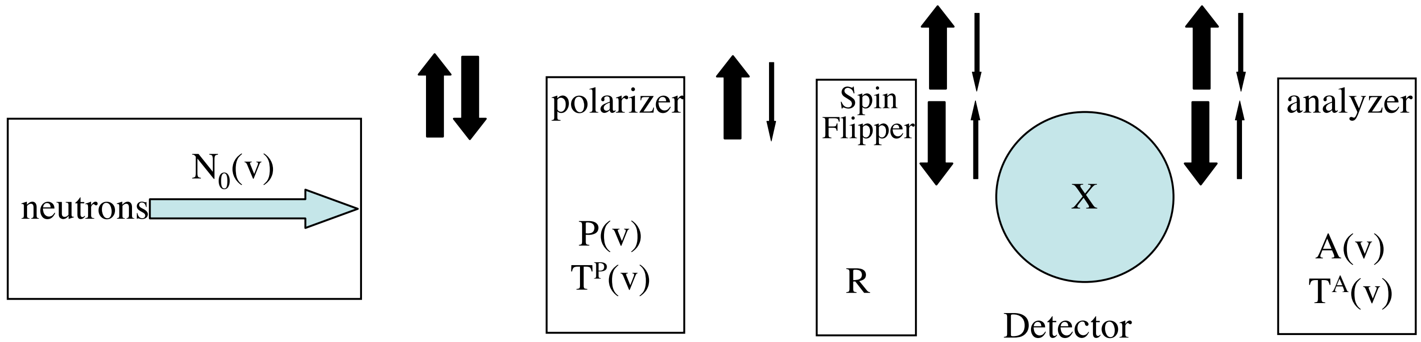
Ideal: $\Gamma_{\pm} = \Sigma \pm \Delta$

Polarizer $P = \dfrac{N_+ - N_-}{N_+ + N_-} = \rho_+ - \rho_- = \dfrac{T_+ - T_-}{T_+ + T_-}$

Flipper: $R^u = 1$ (unflipped); $R^f = \cos\theta \approx -1$ (flipped)

Analyzer $A = \dfrac{T^A_+ - T^A}{T^A_+ + T^A_-}$

Neutron Polarization and Polarimetry



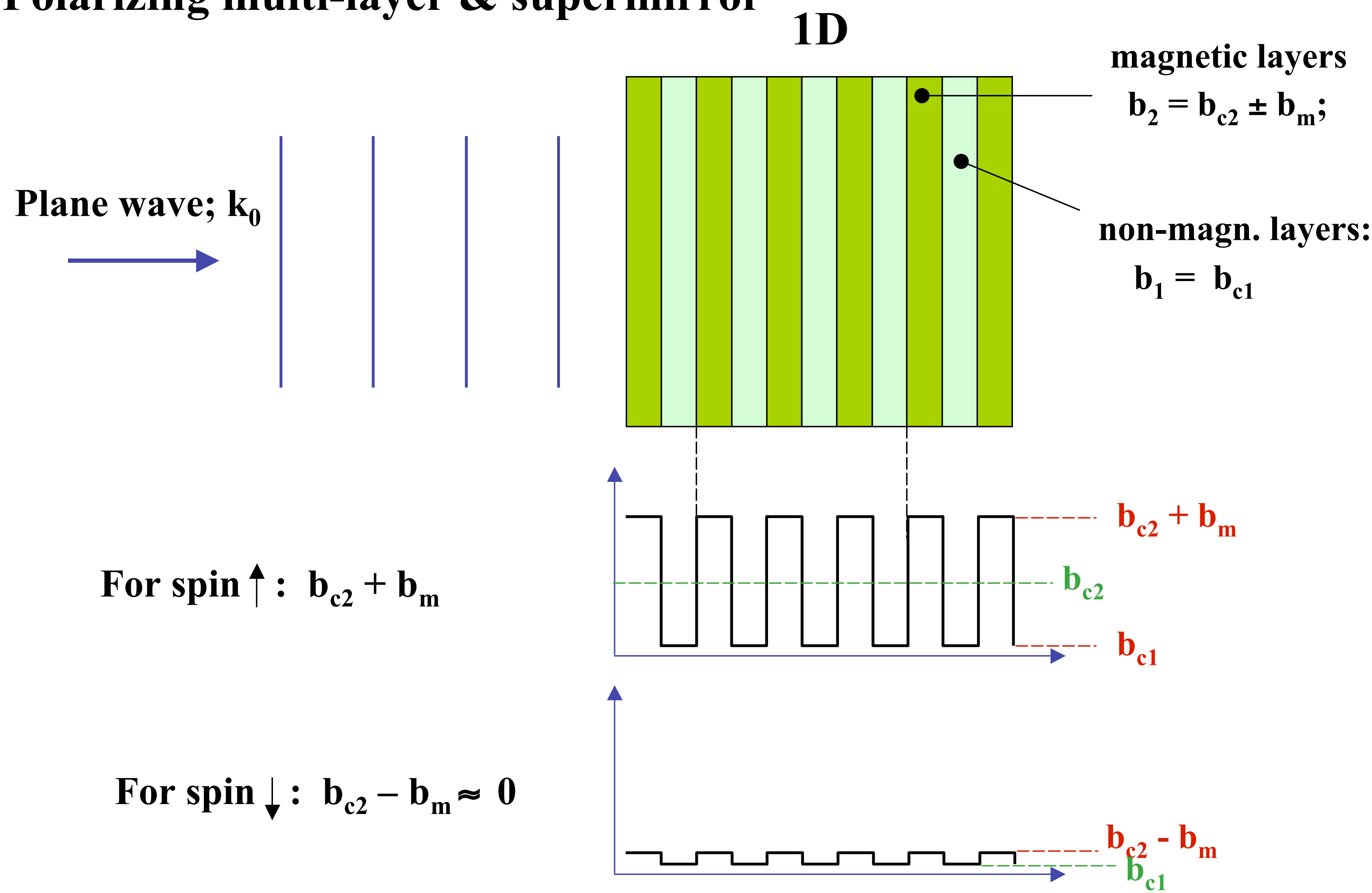
Polarizers: transmit spin-up and down differently

Stern Gerlach, magnetic crystals, spin dependent supermirrors; polarized nuclei (scattering or absorption)

Figure of merit: $\frac{1}{\sigma^2} \propto TP^2$

P/A	P_n (5Å)	T_n	P^2T	features
PSM	99.x%	10%	0.1	fixed; limited λ bite
^{3}He (60%)	80%	30%	0.2	flip P_3; P_3 varies

Polarizing multi-layer & supermirror



- F. Mezei; Commun.Phys.1(1976)81; + Corrigen. : Commun.Phys.2(1977)41; (first paper)
- J. Hayter, A Mook: J. Appl. Cryst.: 22(1989)35; (used for supermirror production)

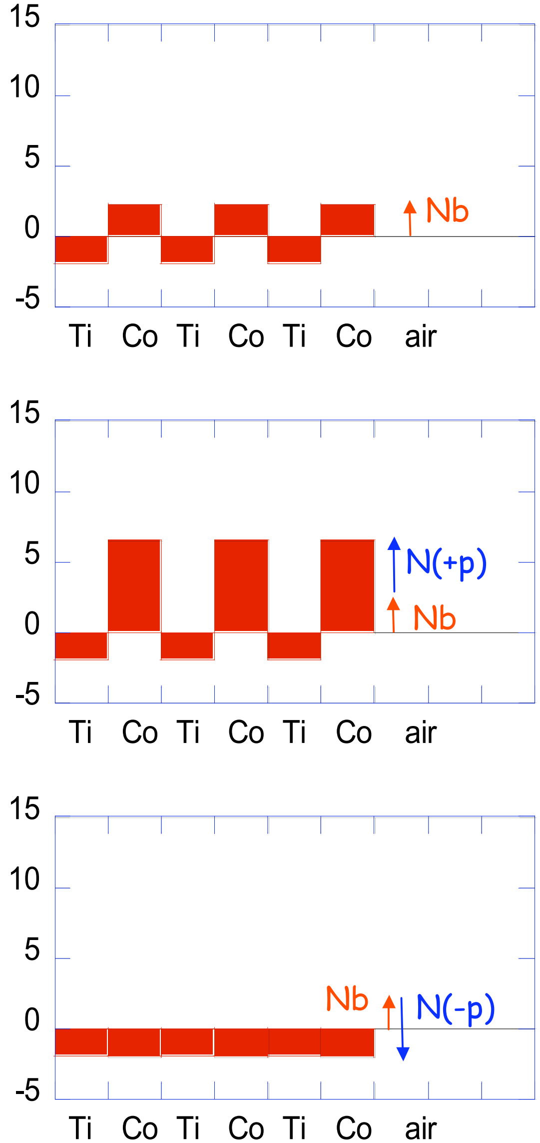
Coherent Scattering Lengths

$$b = \frac{A+1}{A} a$$

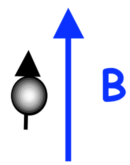
element	b (fm)
H	-3.74
Be	7.79
C	6.65
Al	3.45
Si	4.15
Ti	-3.44
Fe	9.45
Co	2.49
Ni/^{58}Ni	10.3/14.4
Cu/^{65}Cu	7.72/10.6
Cd	4.87-0.7i

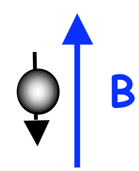
$$n = \sqrt{1 - \lambda^2 \frac{Nb}{2\pi} \pm \lambda^2 \mu B \frac{2m}{(2\pi\hbar)^2}}$$

Co/Ti polarising supermirrors; K. Andersen, ILL

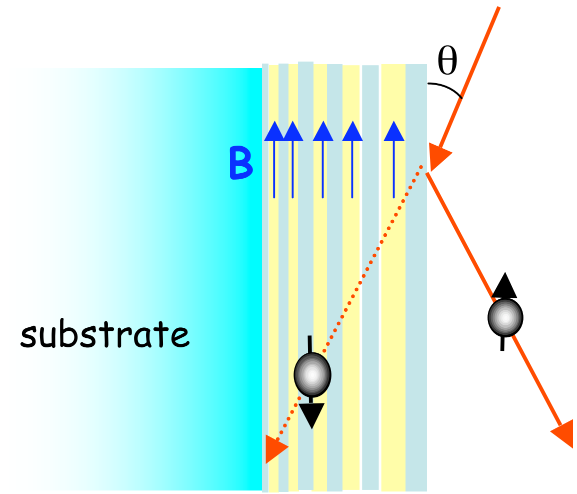


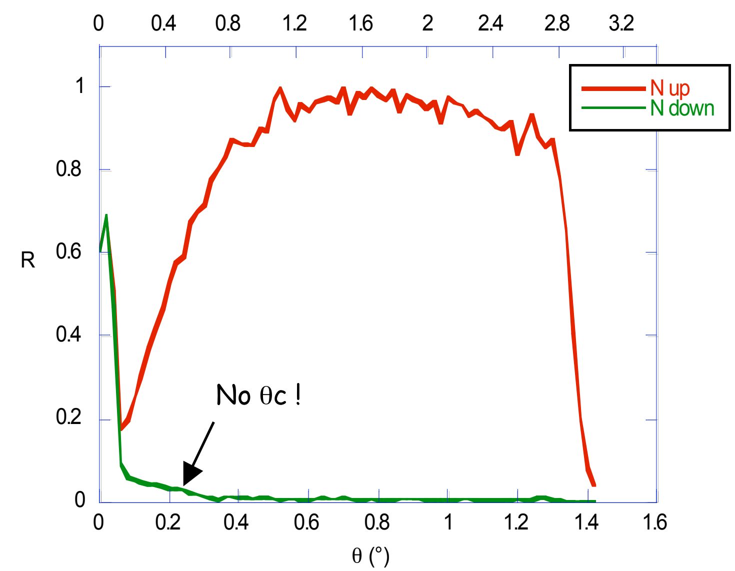
No magnetic field



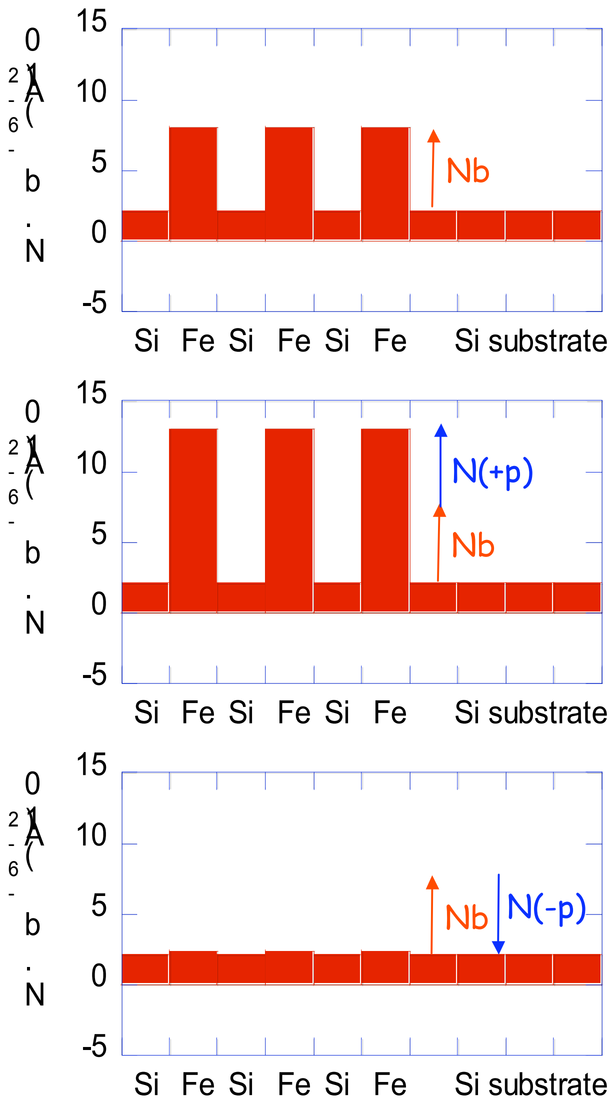


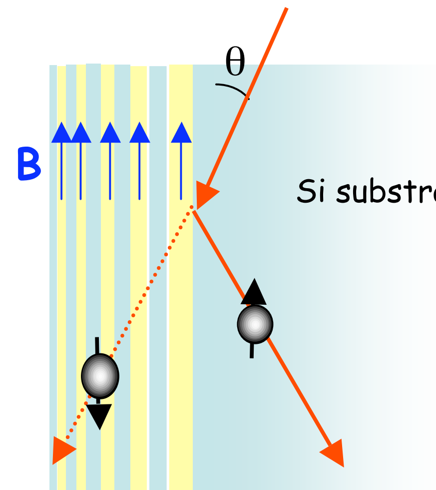
$$n^2 = 1 - \frac{\lambda^2 Nb}{2\pi} > 1$$



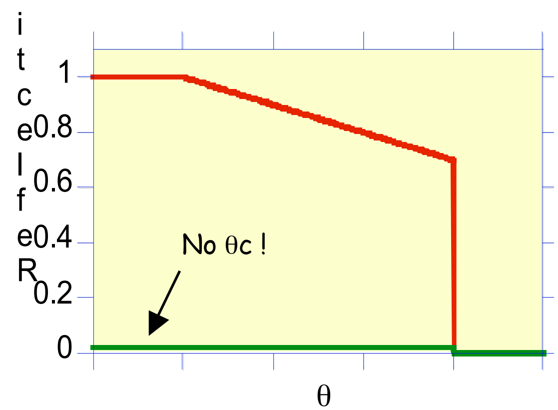


Fe/Si polarising supermirrors; K. Andersen, ILL





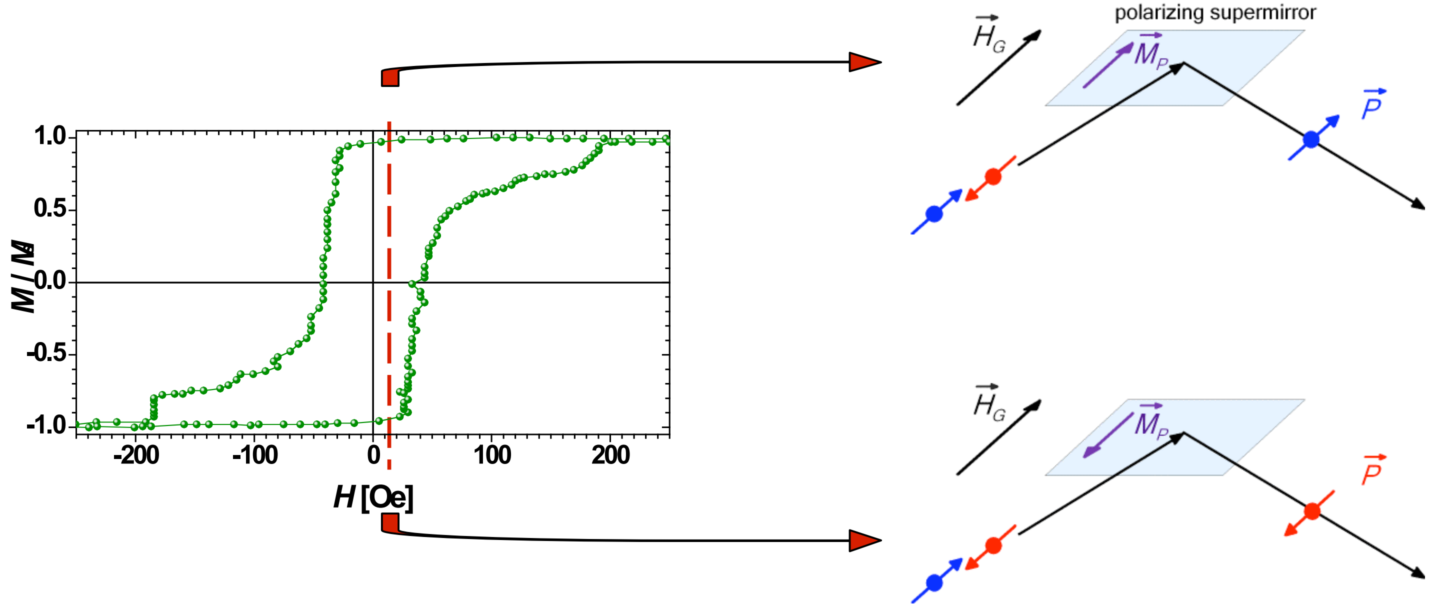
λ=3Å : l_{abs} = 70cm



‘remanent’ polarizing supermirrors - reversible neutorn spin

magnetic anisotropy

high remanence

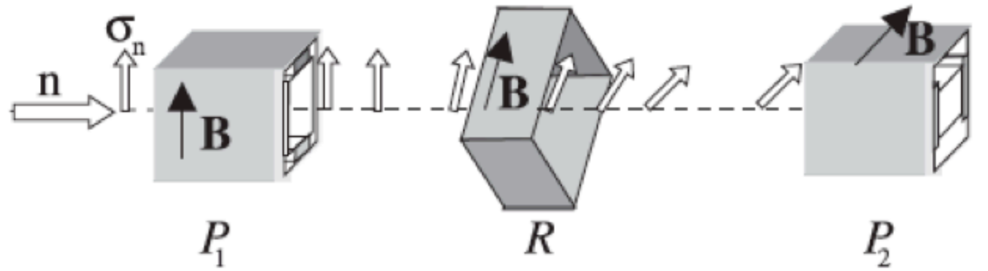


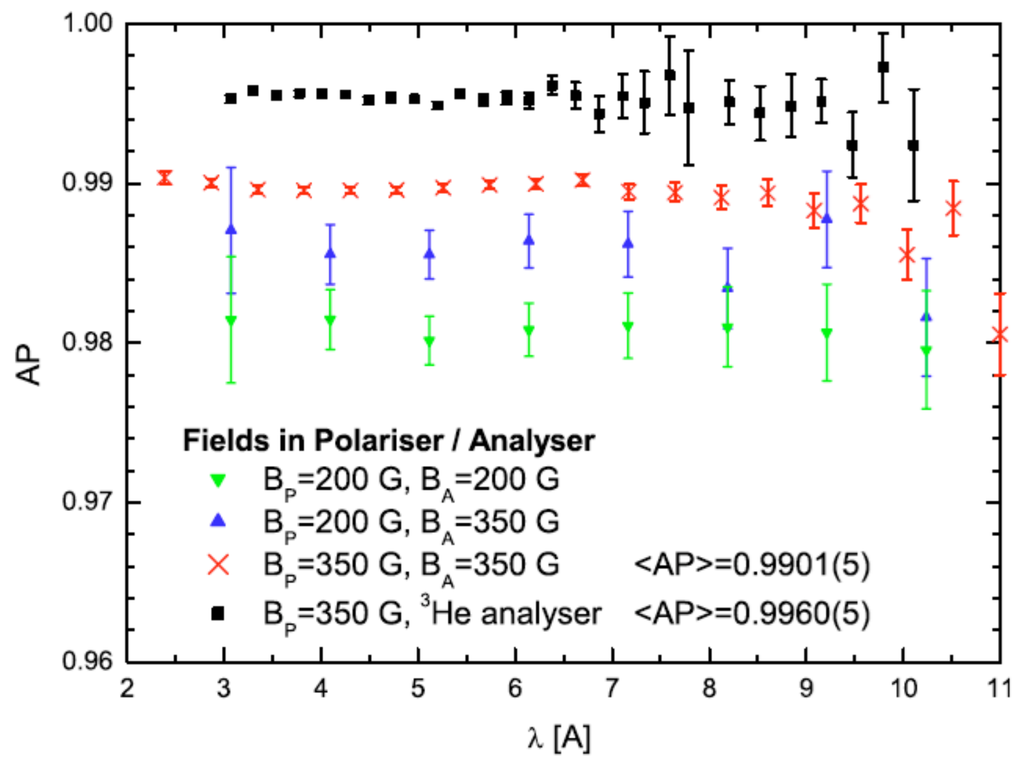
guide field to maintain neutron polarization ≈ 10 G

switching of polarizer/analyzer magnetization ⇒ short field pulse ≈ 300 G

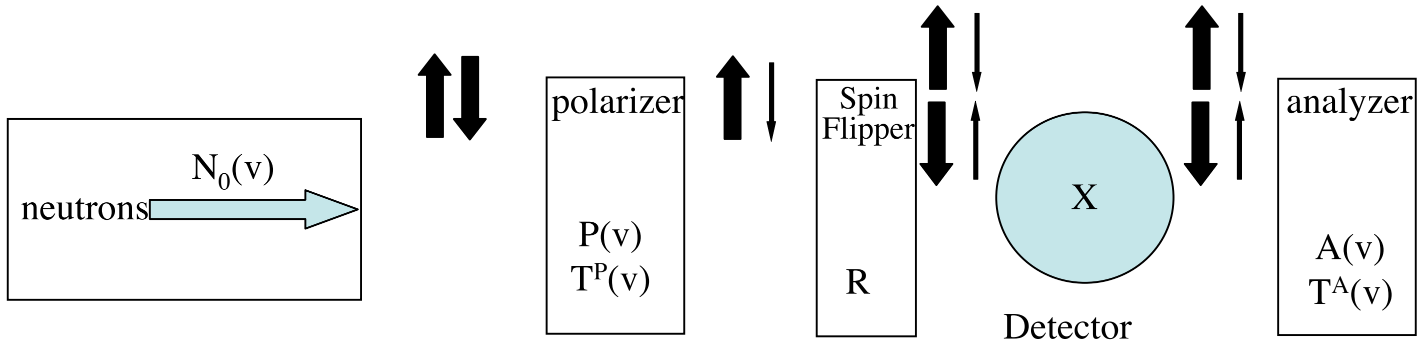
Crossed Supermirror

Kreuz et al./ILL+UM

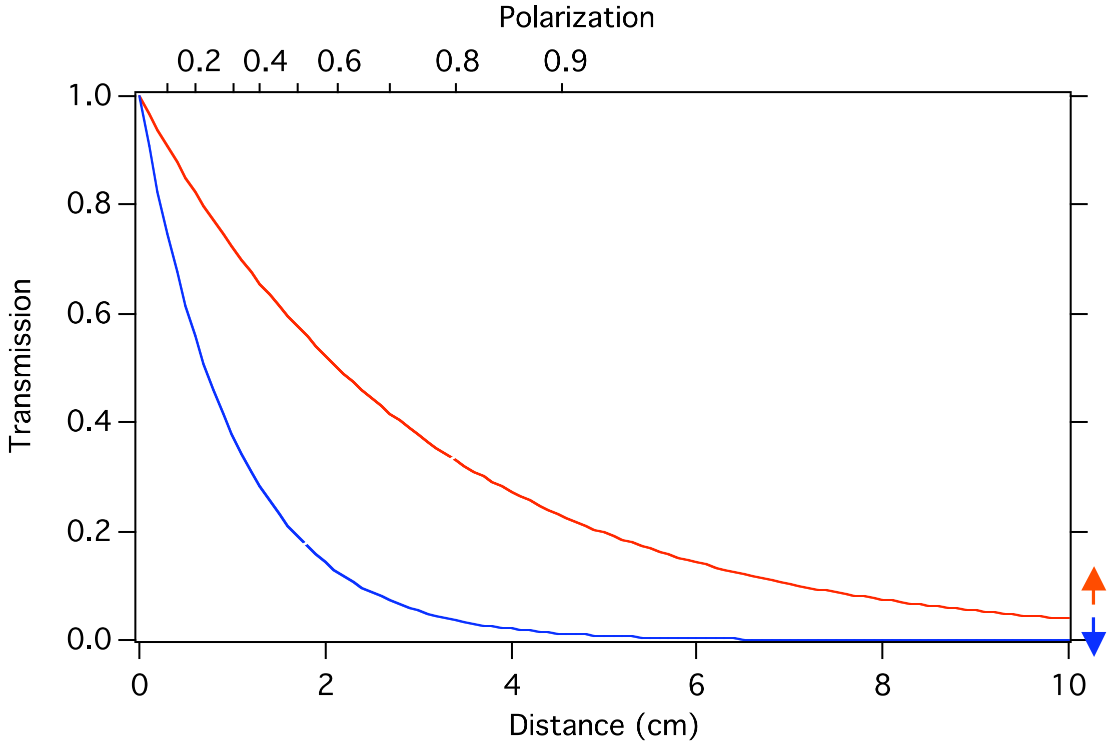




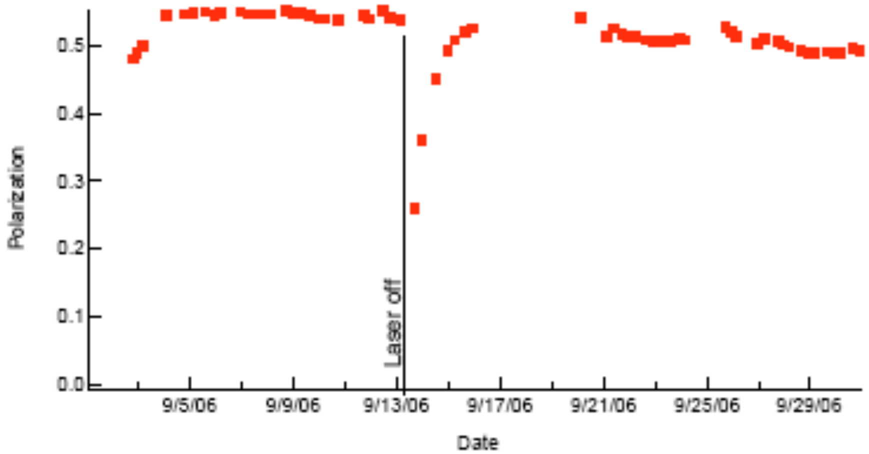
^{3}He spin filter: $n + ^3He \rightarrow ^1H + ^3H$

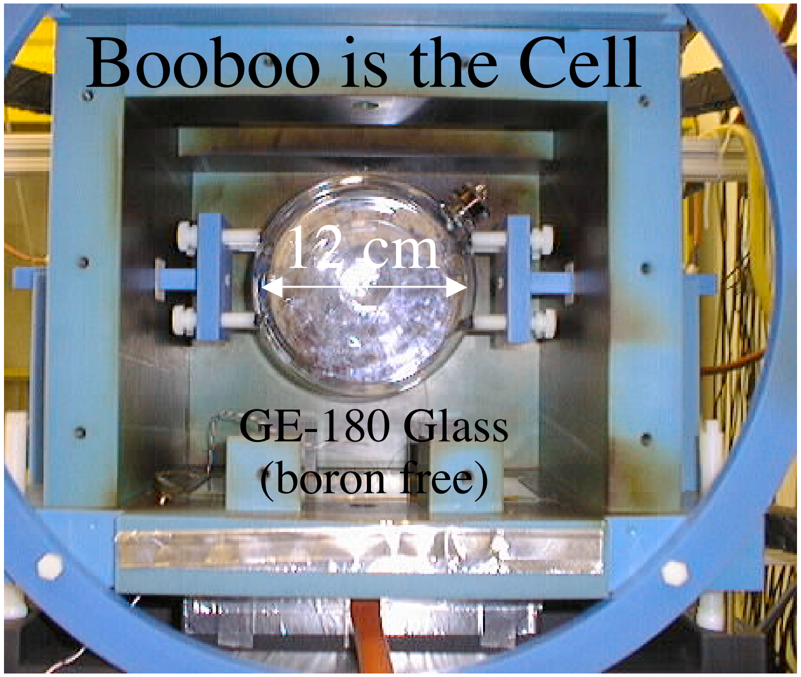


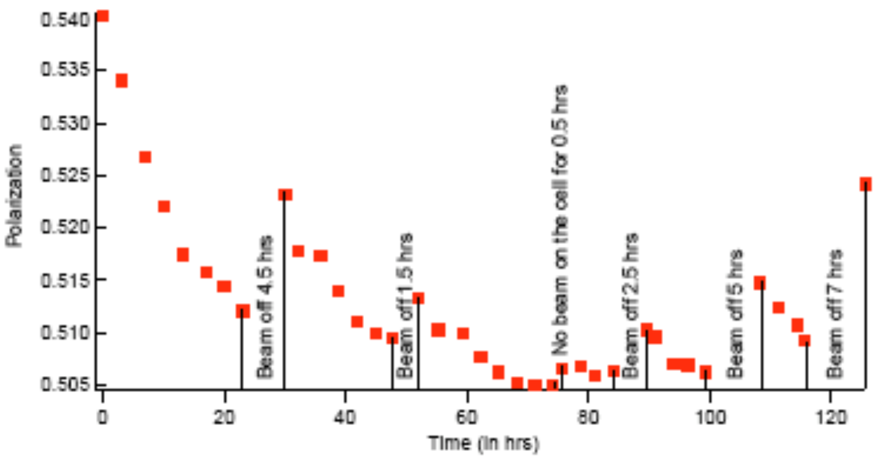
$$\sigma(J=0)=5333\,\frac{\lambda}{\lambda_0}\,b;\ \sigma(J=1)\approx 0\quad(\lambda_0=1.8\ \text{Å})$$



^{3}He spin filter

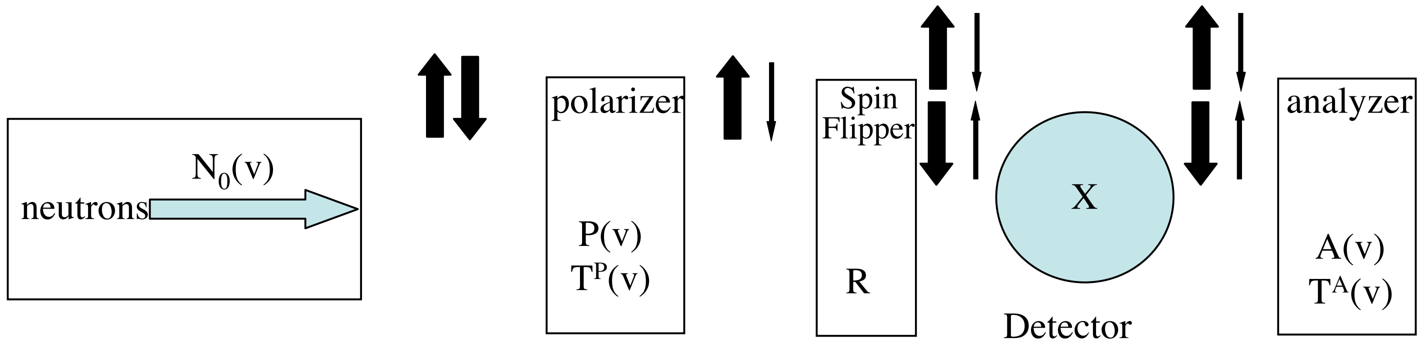




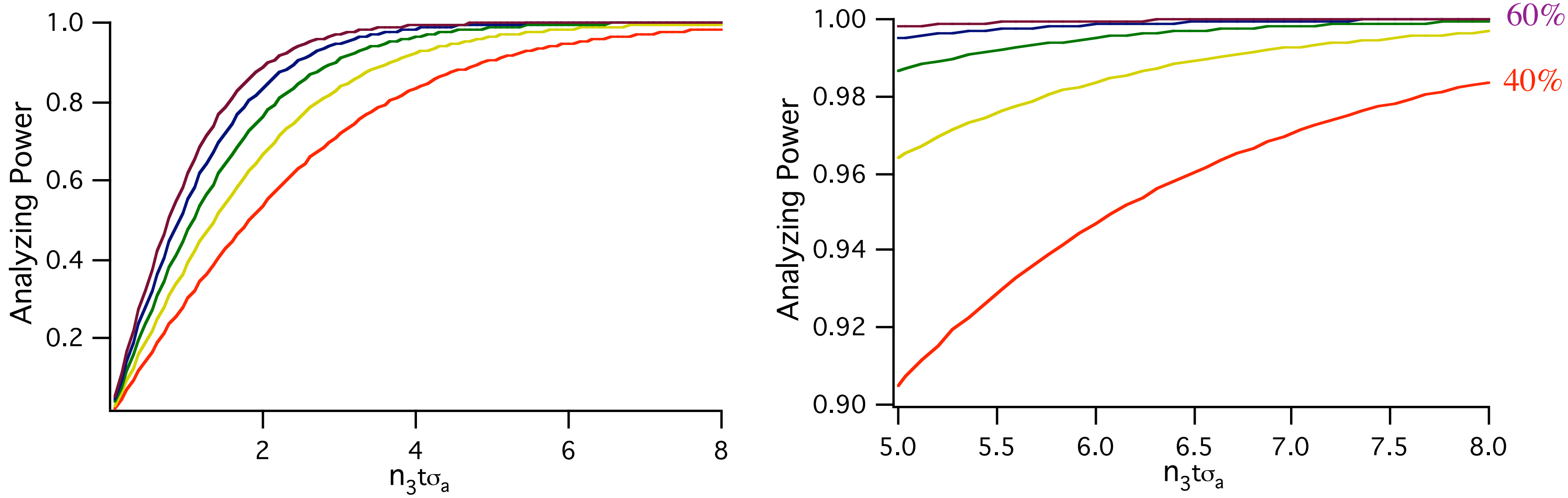


Cells get "milky" deposit
Evidence of Rb depolarization

Neutron Polarimetry

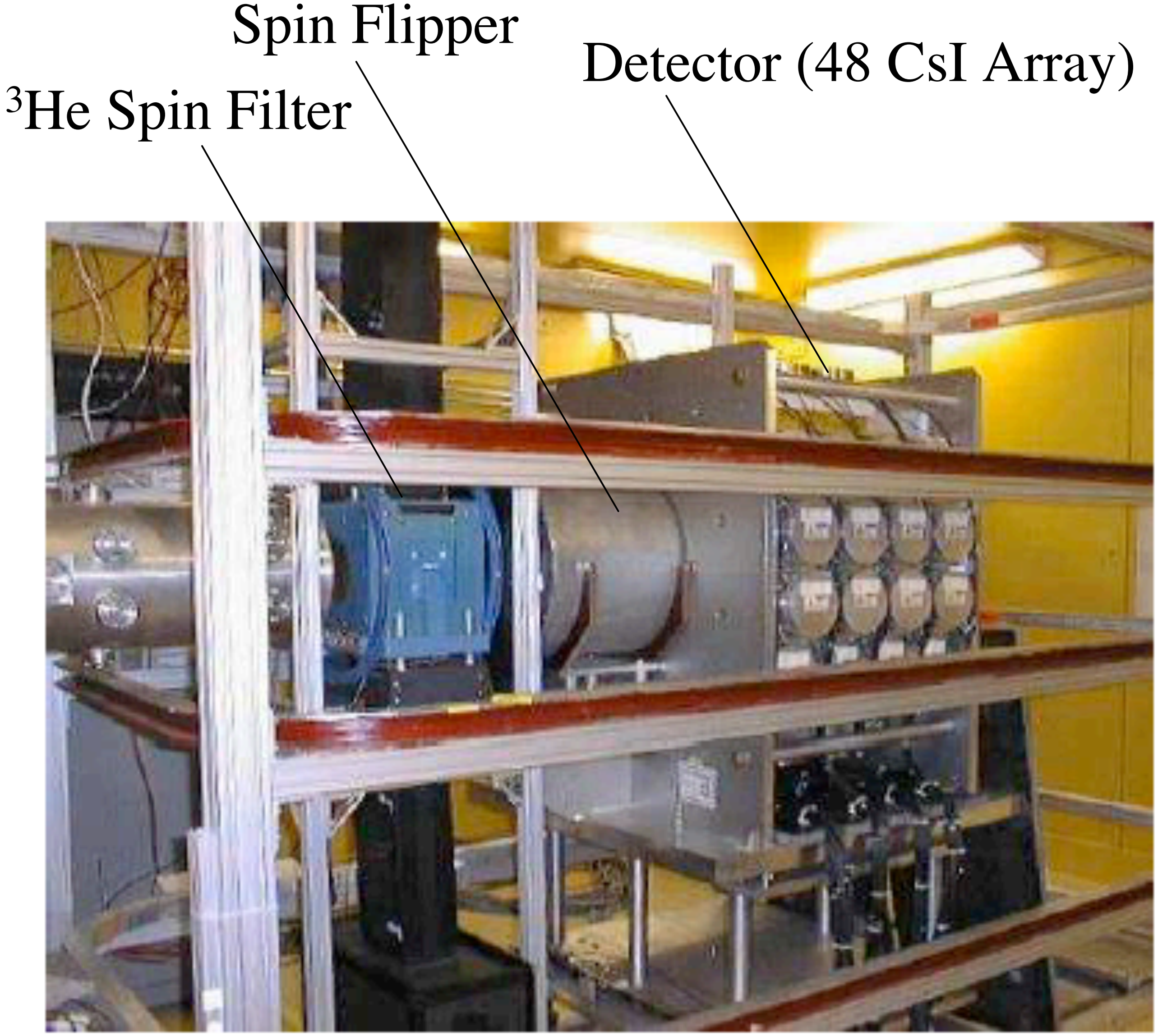


λ=5 Å



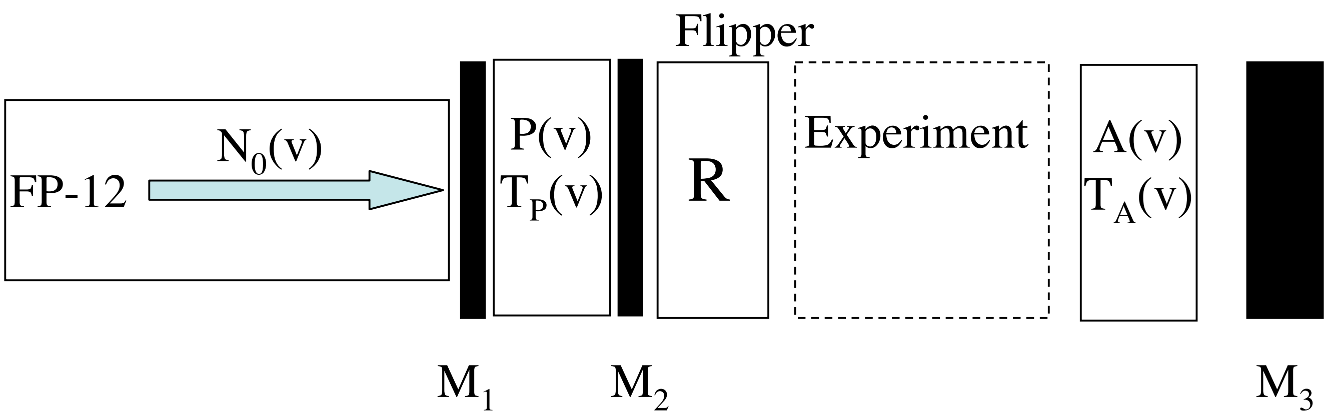
Opaque Analyzer: For $n_3 t\sigma_a$ large enough (1-A) small enough

Can characterize Polarizer AND Flipper



n+p —> d + γ (LANSCE & SNS)

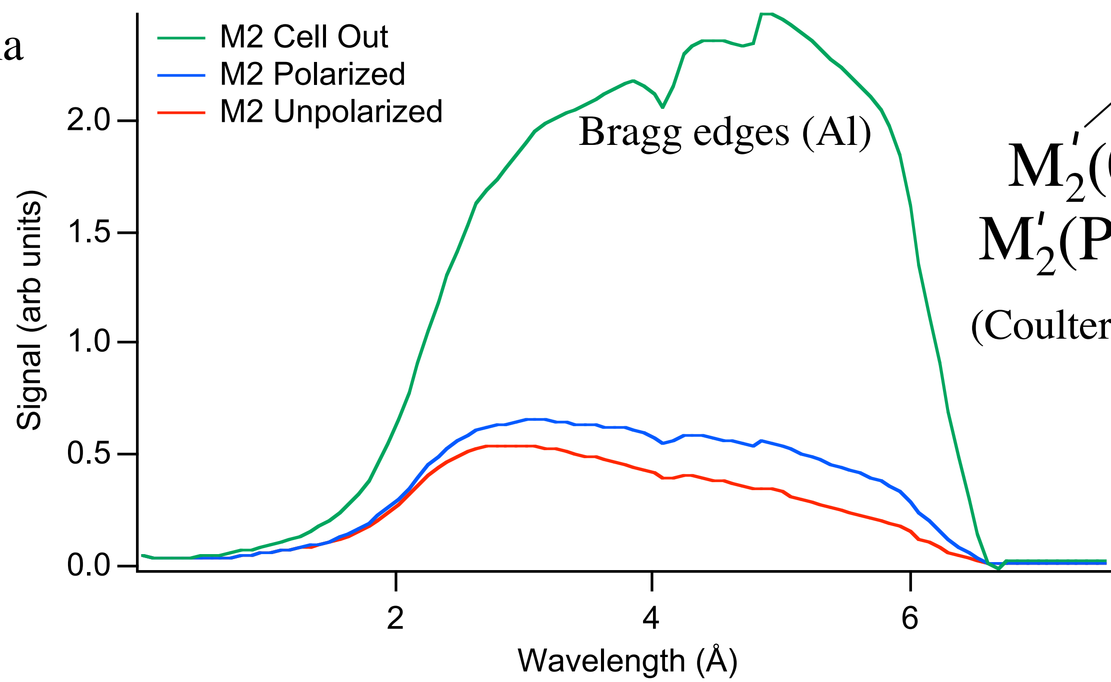
Pulsed Beam Neutron Polarimetry



$$T_{A/P}=T_0\cosh \alpha_{A/P}P_{A/P}$$
$$T_0=\exp(-\alpha_{A/P}-\alpha_{gl}-\ldots)$$
$$P/A=\tanh \alpha_{A/P}P_{A/P}$$

$$M_1= N_0\varepsilon_1+B_1 \qquad M_2= N_0T_1T_P\,\varepsilon_2+B_2$$

npdgamma
FP12

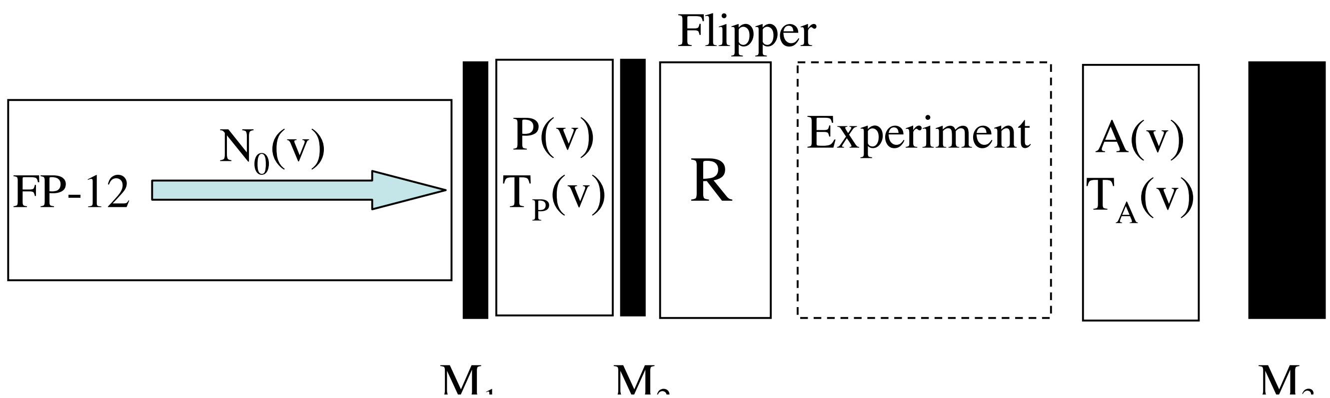


normalized to M1

$$M_2'(0)/M_2'(out) = T_0$$
$$M_2'(P)/M_2'(0) = \cosh \alpha_P P_P$$

(Coulter et al. NIMA ***288***, 463 (1989)

Pulsed Beam Neutron Polarimetry

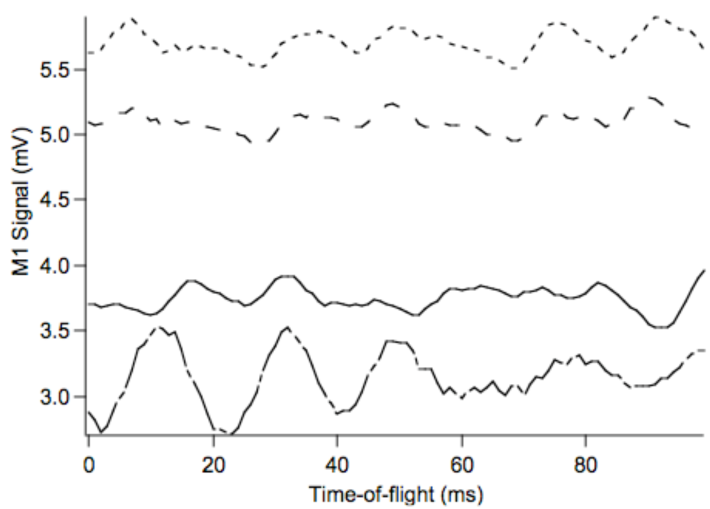


$$T_{A/P}=T_0\cosh \alpha_{A/P}P_{A/P}$$

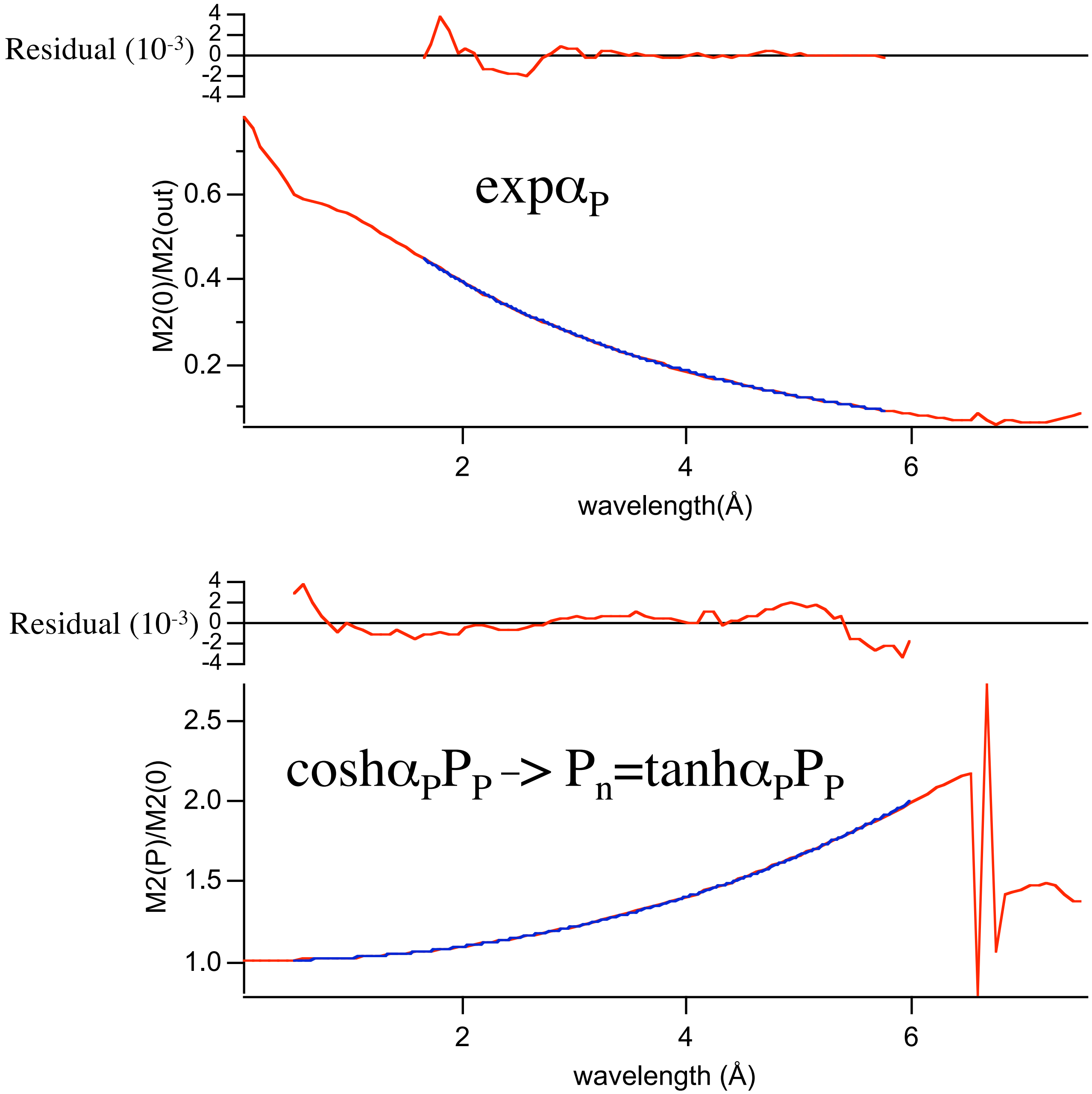
$$T_0=\exp(-\alpha_{A/P}-\alpha_{gl}-\ldots)$$

$$P/A=\tanh \alpha_{A/P}P_{A/P}$$

npdgamma
FP12



Offsets/Backgrounds



Residual (10^{-3})
4
2
0
-2
-4
M2(0)/M2(out)
0.6
0.4
0.2
$\exp\alpha_P$
2
4
6
wavelength(Å)
Residual (10^{-3})
4
2
0
-2
-4
M2(P)/M2(0)
2.5
2.0
1.5
1.0
$\cosh\alpha_P P_P$ -> $P_n=\tanh\alpha_P P_P$
2
4
6
wavelength (Å)

Spin Transport and Spin Flippers

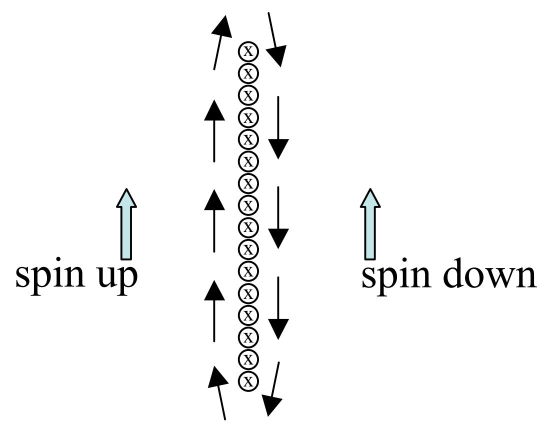
$$\frac{d\vec{s}}{dt} = \gamma\vec{s} \times \vec{B}$$

e.g. $\vec{B} = B_0\hat{z}$, and $\vec{s} = \frac{\hbar}{2}\hat{z}$ so $\frac{d\vec{s}}{dt} = 0$ as long as $\frac{d\vec{B}}{dt} = 0$

$\frac{d\vec{B}}{dt} << \gamma B^2$ is sufficient (adiabatic limit)

A spin in free space is hard to "depolarize" or flip

Exceptions: "diabatic", oscillating fields



Current sheet spin flipper (also Meissner shield)

Resonant Spin Flippers

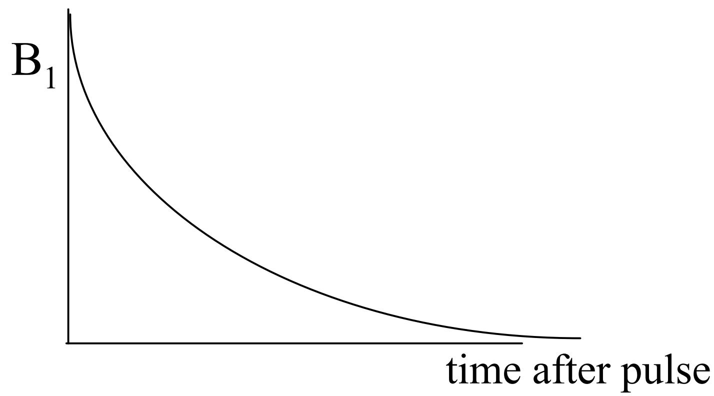
$$\frac{d\vec{s}}{dt} = \gamma \vec{s} \times \vec{B}$$

$$\vec{B} = B_0 \hat{z} + B_1 \cos \omega t \hat{x}$$

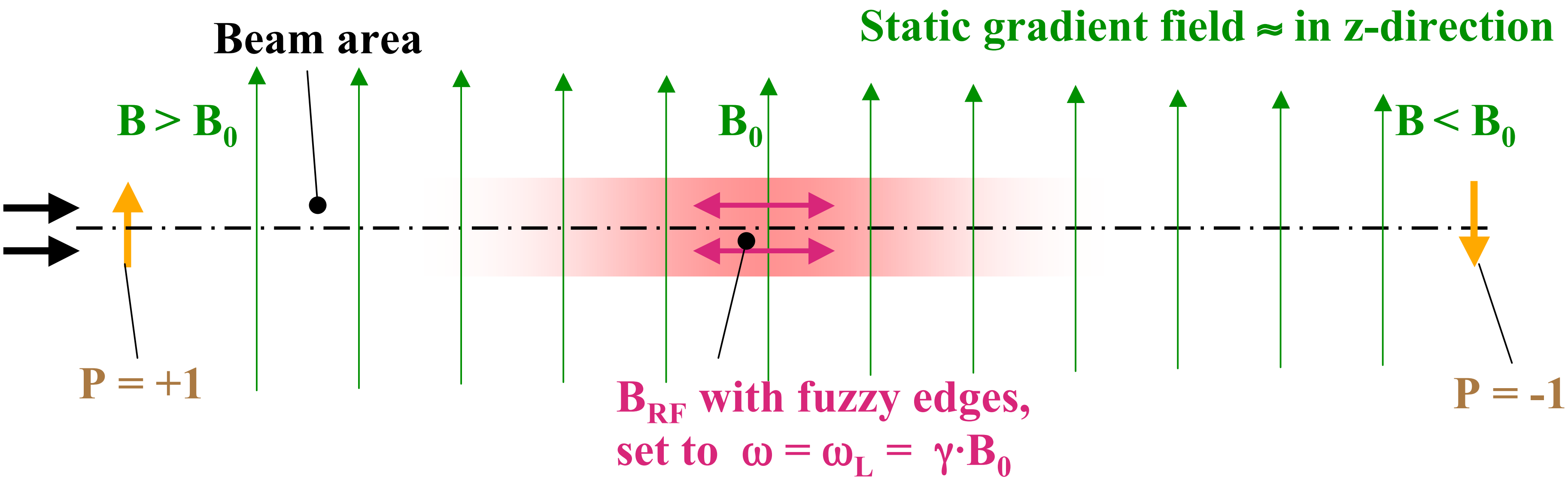
$$\frac{ds_z}{dt} = \gamma B_1$$

Spin flip: $\gamma B_1 t = \pi$ (π pulse)

npdgamma spin flipper: t=L/v



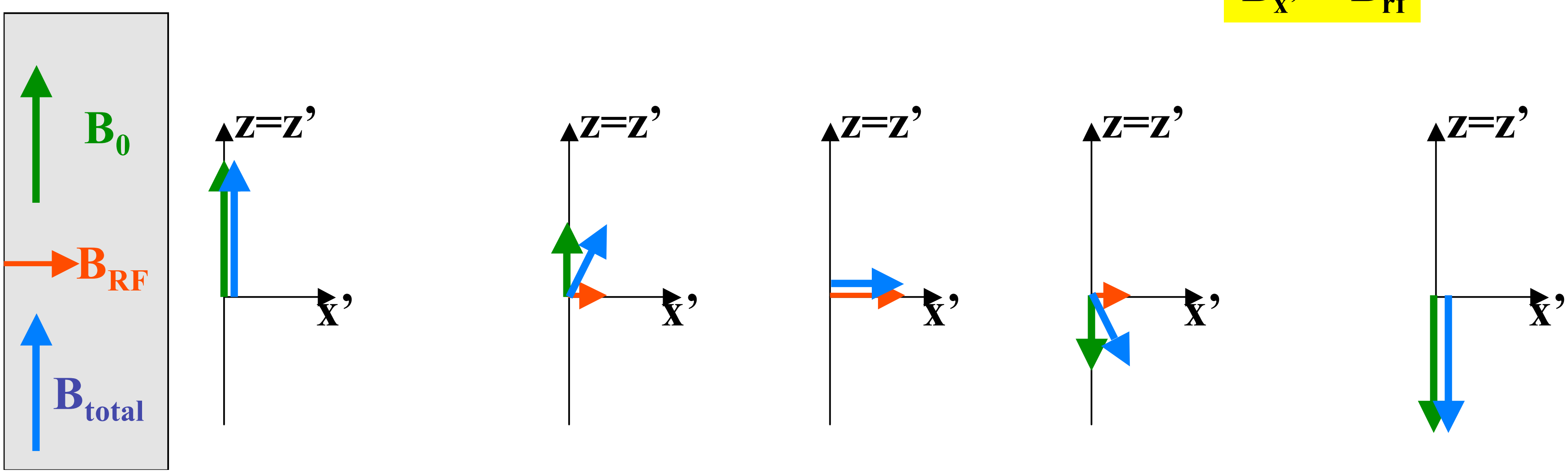
The fast adiabatic spin flipper



Spin turn in a frame [‘] rotating with ω_L around the z-axis:

$$B' = B - \omega_L/\gamma = B - B_0 \;;$$

$$B_{x'} = B_{rf}$$



The fast adiabatic spin flipper (time domain)

$$i\hbar\frac{d\psi}{dt} = \left(-\vec{\mu}\cdot\vec{B}\right)\psi = -\left(\frac{\mu_+ B_- + \mu_- B_+}{2} + \mu_z B_z\right)\psi$$

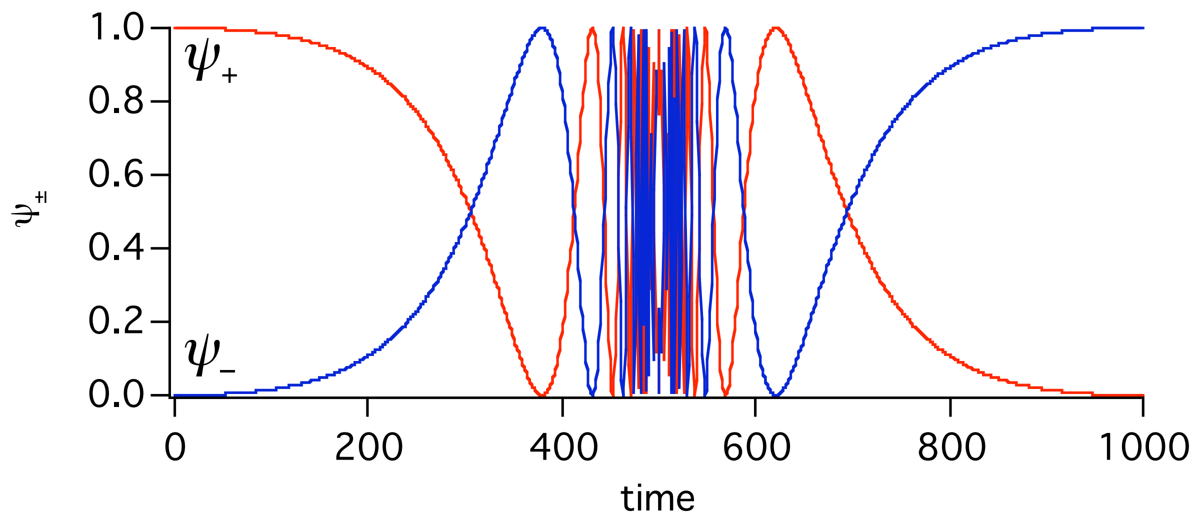
$$\psi = \begin{bmatrix}\psi_+ \\ \psi_-\end{bmatrix} \qquad B_\pm = \frac{B_1}{2}e^{\pm i\omega_0 t}$$

$$B_z = \beta_0 t$$

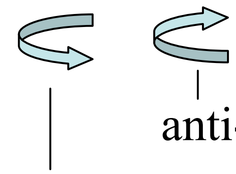
$$i\hbar\frac{d}{dt}\begin{bmatrix}\psi_+ \\ \psi_-\end{bmatrix} = \begin{bmatrix}\mu_z\beta_0 t & \frac{B_1}{2}e^{i\omega_0 t} \\ \frac{B_1}{2}e^{-i\omega_0 t} & \mu_z\beta_0 t\end{bmatrix}\begin{bmatrix}\psi_+ \\ \psi_-\end{bmatrix}$$

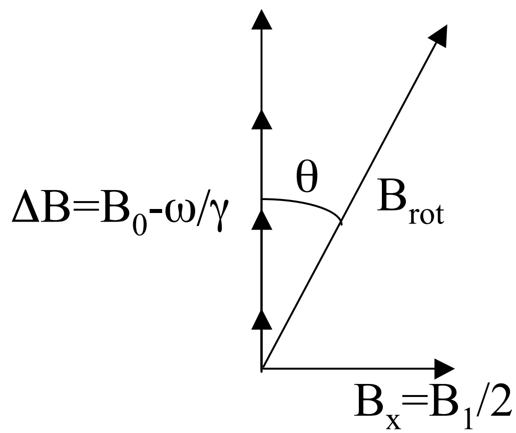
Feynman lectures vol. III

$$i\hbar\frac{d\psi_+}{dt} = \mu_z\beta_0 t\psi_+ + \frac{B_1}{2}e^{i\omega_0 t}\psi_- \qquad i\hbar\frac{d\psi_-}{dt} = \frac{B_1}{2}e^{-i\omega_0 t}\psi_+ + \mu_z\beta_0 t\psi_-$$



Rotating frame (wave) picture: $\cos\omega t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$



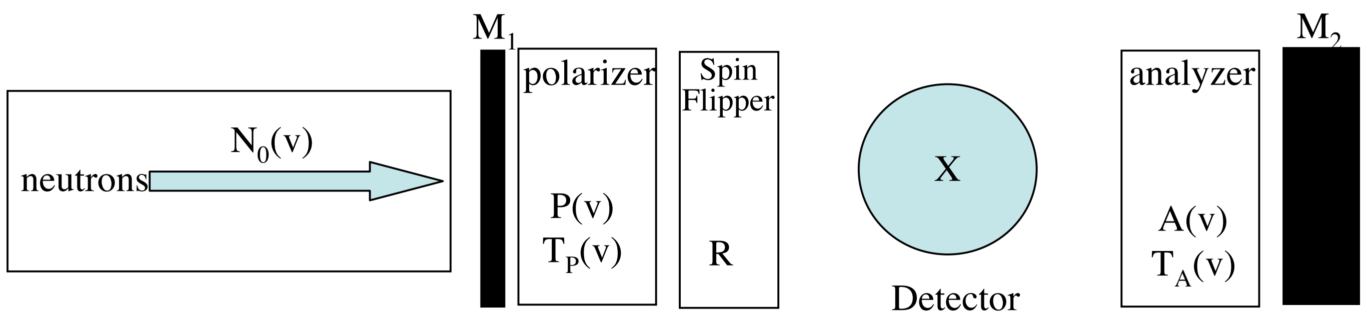


AFP: Spin follows B_{rot} adiabatically $\left(\frac{d\theta}{dt} << \gamma B_1\right)$

Resonant flipper: $\Delta B=0$

Spin rotates around B_x (Rabi frequency $\omega_R = \gamma \frac{B_1}{2}$)

From experiment to physics



$$\frac{N_+ - N_-}{N_+ + N_-} = \mathbf{C}\, P_n\, \mathcal{A}\, \mathcal{F}\, (1-f) + A_{false}$$

- P_n: neutron polarization
- $\mathcal{A}$: analyzing power
- $\mathcal{F}$: spin flip efficacy
- f: background