Neutron Optics and Polarization

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With assistance from notes of R. Gähler; ILL Grenoble

1. Neutron waves
2. Neutron guides
3. Supermirrors
   break
4. Neutron polarization
5. Neutron polarimetry
6. Neutron spin transport/flipping

General references:
V.F. Sears, Neutron Optics, Oxford 1989
Rauch and Werner, Neutron Interferometry, Oxford 2000
Fermi: Nuclear Physics (notes by Orear et al. U. Chicago Press - 1949)
QM Text (e.g. Griffiths)
Optics

Optics: the behavior of light (waves) interacting with matter

Waves characterized by wavelength $\lambda$

Matter characterized by permeability $\kappa$, susceptibility $\mu$, dissipation ($\rho/\sigma$)

Interaction characterized by $n$ (index of refraction); $\delta$ (skin depth)

Useful when $\lambda >> a$ (atomic spacing)
deBroglie: massive particles behave as waves

\[ k = \frac{2\pi}{\lambda} = \frac{p}{\hbar} \]

\[ k_B T = \frac{p^2 c^2}{2mc^2} \quad \lambda^2 = \frac{4\pi^2 (\hbar c)^2}{2mc^2 k_B T} \]

\[ mc^2 = 939.6 \text{ MeV} \]

\[ \hbar c = 197.3 \text{ MeV-fm} = 1973 \text{ eV-Å} \]

\[ v = 2200 \text{ m/s} \quad \lambda = 1.8 \text{ Å} \]  
(thermal neutrons - 300° K)

Note also: \( \lambda \propto \sqrt{\frac{1}{T}} \)
Wave Properties

- **Polarization**
- **Reflection** \( i = r \) (angle of incidence = angle of reflection)
- **Refraction** \( n_i \sin i = n_t \sin t \)
- **Interference**
- **Superposition**
- **Diffraction** \( m \lambda = W \sin \theta \)

Vertically polarized light

\[ c_0 / c_0 = 2.97 \times 10^8 \text{ m/s} \]
The wave equations for light and matter waves in vacuum:

\[ k = \frac{2\pi}{\lambda} = \frac{p}{\hbar} \]

**EM wave equation** (\( \vec{E}, \vec{B} \))

Time dependent:

\[ \nabla^2\Psi - \frac{1}{c^2} \frac{\partial^2\Psi}{\partial t^2} = 0 \]

\[ \Psi(\vec{r}, t) = a_k e^{i(\vec{k} \cdot \vec{r} - \omega_k t)} \]

**Schrödinger equation**

\[ \nabla^2\Psi + \frac{2i}{\hbar} m \frac{\partial\Psi}{\partial t} = 0 \]

**Helmholtz equation:**

\[ \nabla^2\Psi(\vec{r}) + k^2\Psi(\vec{r}) = 0 \]

**Dispersion relations:**

\[ k^2 = \frac{E^2}{(\hbar c)^2} \]

\[ v_{ph} = \frac{\omega}{k} = c \]

\[ (E = \hbar \omega) \]

**Phase velocity:**

\[ k^2 = \frac{2mE}{\hbar^2} \]

\[ v_{ph} = \frac{\omega}{k} \sqrt{1 + \frac{m^2 c^2}{p^2}} \approx \frac{\omega}{c} \left(1 + \frac{m^2 c^4}{p^2} \right) \]

\[ (E = \sqrt{p^2 c^2 + m^2 c^4}) \]
Interactions $V(r)$

\[ \nabla^2 \Psi(\vec{r}) + \frac{2m}{\hbar^2} [E - V(\vec{r})] \Psi(r) = 0 \]

Time independent

Schroedinger equation

\[ \Psi(\vec{r}) = e^{ikz} + \frac{f(\theta)}{r} e^{ik \cdot \vec{r}} \]

Incoming plane wave

Outgoing spherical wave

\[ f(\theta) = \frac{1}{2ik} \sum_l (2l + 1)[e^{2i\delta_l} - 1]P_l(\cos \theta) \]

Partial waves

\[ f(\theta) = \frac{1}{2ik} [e^{2i\delta_0} - 1] = \frac{1}{[k \cot \delta_0 - ik]} \]

s-wave scattering

\[ \delta_0 = -kr_o \quad V(r)=0 \text{ at } r_o \]

\[ a=-\frac{\delta_0}{k}: \text{scattering length} \]

\[ (-5 \text{ fm} < a < 15 \text{ fm}) \]

\[ ka=-\delta_0 \sim 10^{-4} \]

\[ f(\theta) = -a + ika^2 + O(k^2) \]

\[ f(\theta) \approx -a \]
Coherent Scattering Lengths

\[ b = \frac{A+1}{A} a \]

<table>
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\[ f(\theta) = \frac{1}{2ik} \left[ e^{2i\delta_0} - 1 \right] \quad \delta_0 = -ka \]
Index of refraction

\[ \nabla^2 \Psi(\vec{r}) + \frac{2m}{\hbar^2} \left[ E - V(\vec{r}) \right] \Psi(r) = 0 \]

Time independent Schroedinger equation

\[ \nabla^2 \Psi(\vec{r}) + K^2 \Psi(\vec{r}) = 0 \]

\[ K^2 = \frac{2m}{\hbar^2} \left[ E - V(\vec{r}) \right] \]

For light: \( c \Rightarrow \frac{c}{n} = \frac{K}{k} \)

\[ n(\vec{r}) = \sqrt{1 - \frac{V(\vec{r})}{E}} \]  
In general, \( n \) is a tensor, i.e. \( V(r) \) depends on propagation direction

\[ V(\vec{r}) = \sum_i \frac{2\pi \hbar^2}{m} b \delta^3(\vec{r} - \vec{r}_i) \]
Fermi Pseudopotential

(n can also be \(<1\) (attractive) and \( ii \) complex (absorption or incoherent scattering))
Index of refraction

\[ n(\vec{r}) = \sqrt{1 - \frac{V(\vec{r})}{E}} \]

\[ V_{nuc}(\vec{r}) = \sum_i \frac{2\pi\hbar^2}{m} b\delta^3(\vec{r} - \vec{r}_i) \approx \frac{2\pi\hbar^2}{m} bN \]

\[ V_{mag}(\vec{r}) = -\vec{\mu} \cdot \vec{B}_{eff} \]

Materials with Fe, Ni, Co have \( B_{eff} \) s.t. \( V_{mag} \sim V_{nuc} \)

\[ n_{\pm}(\vec{r}) = \sqrt{1 - \frac{V_{nuc}(\vec{r}) + \vec{\mu} \cdot \vec{B}_{eff}}{E}} \]

Note: neutron magnetic moment \( \mu \) is negative, i.e. “spin up” has positive \( V_{mag} \).

(n can also be \( i \) )\( >1 \)(attractive) ; \( ii \) complex (absorption or incoherent scattering)
Notes

\[ n = \sqrt{1 - \frac{\lambda^2 Nb}{2\pi}} \]

\(V(r)\) is generally positive and \(n<1\)

Other interactions

\[ V_{\text{spin-orbit}}(\vec{r}, \vec{p}) = -\frac{1}{m} \vec{\mu} \times (\vec{p} \times \vec{E}) \]

\[ V_{\text{electric}}(\vec{r}) = \hbar c \alpha \sum_i \int \frac{\rho(\vec{r}')}{|\vec{r}_i - \vec{r}'|} d^3 r \]
Neutron Charge Distribution

Slope gives $<r_q^2>$ ($<0$)
2. Neutron guides
Neutron guides: assume no Bragg scattering; absorption negligible;

In case of different atoms $i$, use the weighted average $\langle n_i \cdot b_i \rangle$.

$b$ is generally positive (reflection from edge of square well*) so $n<1$

*see Peshkin & Ringo, Am. J. Phys. 39, 324 (1971)

Neutrons are totally reflected, if $E_\perp < V$

$$ E_\perp = \frac{1}{2} m v_\perp^2 = \frac{\hbar^2 k_\perp^2}{2m} = \frac{2\pi^2 \hbar^2}{m \lambda_\perp^2} $$

$$ k_\perp = k \sin \theta \quad \text{and} \quad \lambda_\perp = \lambda / \sin \theta $$

$$ \frac{2\pi^2 \hbar^2}{m \lambda_\perp^2} < V \quad \text{or} \quad \sin \theta < \sqrt{\frac{mV}{2\pi^2 \hbar^2}} \lambda \quad \Rightarrow \text{critical angle} \ \theta_C $$

For $^{58}\text{Ni}$, $\theta_C / \lambda = 2.03 \text{ mrad} / \text{Å} \ (1.73 \text{ mrad} / \text{Å} \text{ for natural Ni})$: $m=1$
Basic properties of ideal bent guides

\[ \theta_c = \sin^{-1}\left(\sqrt{\frac{mV}{2\pi^2\hbar^2\lambda}}\right) \]

All reflections are assumed to be specular with reflectivity 1 up to a well defined critical angle \( \theta_c \) and with reflectivity 0 above \( \theta_c \).

There are two types of reflections:

- **Zig-zag reflections** (large \( \theta_a \))
- **Garland reflections** (never touching the inner wall) (small \( \theta_a \))

If the max. reflection angle allows only Garland reflections near the outer wall, then the guide is not efficiently ``filled.''

If \( \theta_a \approx \theta_i \) the filling of the guide will be fairly isotropic (many reflections).

After at least one reflection of all neutrons, the angular distribution in the guide is well defined. The angles always repeat.
The Maier-Leibnitz guide formula

\[
y = \rho \cdot \sin(\theta_a - \theta) \approx \rho \cdot (\theta_a - \theta);
\]

\[
x_1 = \rho - \rho \cdot \cos(\theta_a - \theta) \approx \rho / 2 \cdot (\theta_a - \theta)^2;
\]

\[
x_2 = y \cdot \tan \theta \approx y \cdot \theta \approx \rho \cdot (\theta_a - \theta) \cdot \theta;
\]

\[
x = x_1 + x_2 = \rho / 2 \cdot (\theta_a^2 - \theta^2);
\]

\[
\theta^2 = \theta_a^2 - 2x / \rho;
\]
The intensity distribution in a long curved guide is a function of $\lambda$

To calculate the max. transmitted divergence, we choose: $\theta_a = \theta_c = k_\perp/k$;

Plot of $\theta_{\text{max}} = (\theta_c^2 - 2x/\rho)^{\frac{1}{2}}$ as function of $x$ for different $\theta_c$ shows added divergence:

For $\theta = \theta_c$ the inner wall just does not get touched.
In this case the divergence is 0 at the inner wall.
The wavelength $\lambda$, corresponding to this $\theta_c$ is called characteristic wavelength $\lambda^*$.
Filling factor $F$ (intensity ratio curved guide/straight guide):

$$\theta_{max}^2 = \theta_a^2 - 2x / \rho$$

$$F = \frac{1}{a \theta_c} \int_0^{a^*} \theta dx = \frac{1}{a} \int_0^{a^*} \sqrt{1 - \frac{2x}{\rho \theta_c^2}} dx$$

$$\theta^* = (2a/\rho)^{1/2}$$

$$a^* = \gamma_c \rho / 2$$

$$F(\theta_c = \theta^*) = 2/3; \quad F(\theta_c = 2\theta^*) = 0.93; \quad F(\theta_c = \theta^*/2) = 1/6;$$
Multilayer mirrors and supermirrors:
Can we exceed the critical angle of “total external reflection?” $\theta_C = \sin^{-1}\left(\sqrt{\frac{mV}{2\pi^2\hbar^2\lambda}}\right)$

Multilayers provide reflections from multiple interfaces between different indexes of refraction

Reflectivity for normal incidence at one interface
$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

for $n_1 < n_2$

constructive interference for $\lambda = 2n_1d_1\sin\theta_i$
destructive interference for $\lambda = 2n_2d_2\sin\theta_2$

Let $n_1d_1 = n_2d_2$
Pick $n_1 + n_2$ small: only a few layers provides high reflectivity for mirror
FOR CONSTANT $\lambda\sin\theta_i$

Supermirror: vary $nd$ for quasi-continuous $\lambda$ and $\theta_i$
refractive index $n < 1$

total external reflection e.g. Ni $\theta_c = 0.1 \, ^\circ/\text{Å}$

$\lambda = 2nd \sin \theta$

increasing $d$
Multilayer supermirrors: Characterized by $m = \frac{\theta_{\text{mirror}}}{\theta_{\text{Ni}}}$

1.73 mrad/Å

References:
F. Mezei, Novel polarized neutron devices: supermirror and spin component amplifier, Communications on Physics 1, 81, 1976.
layer sequence $\lambda/4$ layer thickness, overlap of superlattice Bragg peaks


high $m$ - value - more angles

$\Rightarrow$ large number of layers, e.g.

$m = 2 \Rightarrow 120$ layers ($R \geq 90\%$)
$m = 3 \Rightarrow 400$ layers ($R \geq 80\%$)
$m = 4 \Rightarrow 1200$ layers ($R \approx 75\%$)
$m = 5 \Rightarrow 2400$ layers ($R \approx 63\%$)

From Swiss Neutronics
Replacing a Ni guide (m=1) by a SM guide (m=2) doubles $k_\perp$ and $\gamma_c$.

Increasing the guide width from $a \Rightarrow 1.5a$ increases $\gamma^*$ by $1.5^{\frac{1}{2}}$ and also increases the direct line of sight $[L_d = (8ap)^{\frac{1}{2}}]$ by $1.5^{\frac{1}{2}}$.

For $\lambda \neq \lambda^*$ the intensity increases by a factor 4.

New characteristic $\gamma^*$ from increase of width

For $\lambda + \lambda^*$ the intensity increases by more than a factor 4 from doubling of $\gamma_c$. [$\gamma_c/2 \Rightarrow \gamma_c$ is shown.]
Guide losses

Reflectivity:

Garland refl.: \( l_g = 2 \rho \gamma \); Zig-zag refl.: \( l_z = \rho (\gamma_a - \gamma_i) \); or \( l_z \approx d/\gamma \);

\[ R^n = (1 - \Delta)^n \approx 1 - n \Delta; \] for \( \Delta \ll 1 \);

for \( R = 0.97 \) and \( L = 1.2 L_0 \); \( n \geq 3 \) (\( \rho = 2700 \text{m}; \gamma = 1/200 \))

\( \Rightarrow \) beam transmission \( f_R \) by reflectivity \( R \): \( < f_R > = 0.9 \); higher for long SM-guides!

\( l = \) mean length for one reflection from side walls;
\( n = \) mean number of reflections; \( R = \) reflectivity; \( L_0 = \) length of free sight; \( L=100\text{m} \)

Alignment errors: Gauss distrib.: \( f(h) = \exp(-h^2/h_f^2) / (\pi^{-1/2} h_f) \)

Transmission \( f_a = 1 - L/L_p \cdot h_f \cdot \pi^{-1/2} / a; \)

For \( L = 1.2 L_0 \); \( a = 3 \text{ cm}, L_p = 1\text{m}, L = 100\text{m} \) and \( h_f = 20 \mu\text{m} \):

\( \Rightarrow \) mean transmission due to alignment errors: \( f_a = 0.96; \)

For guide of 20x3 cm, the necessary precision on top / bottom is 7 times worse (0.14 mm).
\( h_f = \) mean alignment error; \( a = \) guide width (30 mm); \( L_p = \) length of plates;
**Guide losses**

_waviness:_

Let the neutron be reflected under angle $\gamma + \alpha$ instead of $\gamma$. 

$$g(\alpha) = \frac{1}{\sqrt{\pi \cdot \alpha_w^2}} \exp\left(-\frac{\alpha^2}{\alpha_w^2}\right)$$

For 6 reflections, $L = 1.2 L_0$; $k = 0.7k^*$: 

$$f_w = 1 - \alpha_w / \gamma^*;$$

For $\alpha_w = 10^{-4}$, $\gamma^* = 1.7 \cdot 10^{-3}$ [1Å, Ni]: 

$$f_w = 0.94;$$

The outer areas of the intensity distribution $I(\gamma)$ are more affected than the inner ones.

$f_w$: transmission due to waviness; $\alpha_w =$ RMS waviness; $\gamma^*$ = characteristic angle of guide

$\alpha_w = 2 \cdot 10^{-4}$ for the new guides seems acceptable (m=2!) 

The angle $\Delta \alpha$ between the guide sections can be treated as waviness. $\Delta \alpha = 1/27000$ for H2.
Summary of main guide formulas

\[ \gamma = (\gamma_c^2 - 2x/\rho)^{1/2} \]
\[ \gamma = \text{angular width at output of curved guide; } \gamma_c = \text{critical angle of total reflection;} \]
\[ \Rightarrow \gamma^* = (2a/\rho)^{1/2}; \gamma^* = \text{characteristic angle;} \]
\[ \text{conditions: each neutron is at least once reflected at outer surface and reflectivity is step function up to } \gamma_c; \]

\[ \lambda^* = 2\pi (2a/\rho)^{1/2} / k_\perp; \lambda^* = \text{characteristic wavelength;} \]

Flux \( d\phi/d\lambda \) for given source brilliance \( d^2\phi/d\lambda d\Omega \) and distance \( z \) from source:

\[ \frac{d\Phi}{d\lambda} = \int_{\text{source}} \frac{d^2\Phi}{d\lambda d\Omega} \frac{1}{z^2} dF \]

Flux \( d\phi/d\lambda \) in long straight guide for constant source brilliance \( d^2\phi/d\lambda d\Omega \) if angular acceptance in guide \( \gamma_x \gamma_y \) is smaller than angular emittance of source:

\[ \frac{d\Phi}{d\lambda} = \frac{d^2\Phi}{d\lambda d\Omega} \gamma_x \gamma_y \]
Summary of main guide formulas

\[ F = \frac{1}{a} \int_0^{a^*} dx \gamma = \frac{1}{a} \int_0^{a^*} dx \left( 1 - \frac{2x}{\rho \cdot \gamma_c^2} \right)^{1/2} \]

\[ a^* = a \quad \text{for} \ \gamma_c \geq \gamma^*; \]
\[ a^* = \rho \gamma_c^2/2 \quad \text{for} \ \gamma_c \leq \gamma^*; \]

\[ F = \text{filling factor of guide} \]

\[ L_0^2 = 8\rho a; \quad L_0 = \text{direct line of sight of bent guide}; \]

\[ \Delta = a -(L_0/2-dL)^2/2R; \quad \Delta = \text{width of direct sight of bent guide; } dL = \text{missing length to } L_0 \]

\[ x_b = L^2/(2\rho); \quad x_b = \text{lateral deviation from start direction; } L = \text{length of guide}; \]

\[ n_g = L/(2 \rho \gamma); \quad n_z = L/(\rho (\gamma_a - \gamma_i)); \quad \text{or } n_z \approx L \gamma/a; \quad n = \text{number of reflections}; \quad n_g \text{ for Garland}; \quad n_z \text{ for Zig-zag refl.;} \]

\[ f_w = 1 - \alpha_w /\gamma^*; \quad f_w = \text{loss due to waviness; } \alpha_w = \text{RMS value of waviness; } L = 1.2 l_d; \]

\[ f_a = 1 - L/L_p \cdot h_f \cdot \pi^{-1/2}/a; \quad f_a = \text{loss due to steps; } h_f = \text{RMS value of alignment error;} \]

Loss \( V \) in intensity due to gap of length \( L \) for a guide of cross section \( a \times b \):

\[ V = \frac{L \cdot \gamma_c}{2} \left( \frac{1}{a} - \frac{1}{b} \right) - \frac{L^2 \cdot \gamma_c^2}{4ab} \]
Break
Neutron Polarization and Polarimetry

\[
R_{\text{Exp}} = \Sigma (\uparrow + \downarrow) + \Delta (\uparrow - \downarrow) = N_0 T_1 T_2 T_P [\Sigma + \Delta PR]
\]

Ideal: \( \Gamma_\pm = \Sigma \pm \Delta \)

Polarizer \( P = \frac{N_+ - N_-}{N_+ + N_-} = \rho_+ - \rho_- = \frac{T_+ - T_-}{T_+ + T_-} \)

Flipper: \( R^u = 1 \) (unflipped); \( R^f = \cos \theta \approx -1 \) (flipped)

Analyzer \( A = \frac{T^A_+ - T^A_-}{T^A_+ + T^A_-} \)
Neutron Polarization and Polarimetry

Polarizers: transmit spin-up and down differently

Stern Gerlach, magnetic crystals, spin dependent supermirrors; polarized nuclei (scattering or absorption)

Figure of merit: \( \frac{1}{\sigma^2} \propto TP^2 \)

<table>
<thead>
<tr>
<th>P/A</th>
<th>( P_n ) (5Å)</th>
<th>( T_n )</th>
<th>( P^2T )</th>
<th>features</th>
</tr>
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<tbody>
<tr>
<td>PSM</td>
<td>99.x%</td>
<td>10%</td>
<td>0.1</td>
<td>fixed; limited ( \lambda ) bite</td>
</tr>
<tr>
<td>(^3\text{He} ) (60%)</td>
<td>80%</td>
<td>30%</td>
<td>0.2</td>
<td>flip ( P_3 ); ( P_3 ) varies</td>
</tr>
</tbody>
</table>
For spin $\uparrow$: $b_{c2} + b_m$

For spin $\downarrow$: $b_{c2} - b_m \approx 0$

• F. Mezei; Commun.Phys.1(1976)81; + Corrigen. : Commun.Phys.2(1977)41; (first paper)
• J. Hayter, A Mook: J. Appl. Cryst.: 22(1989)35; (used for supermirror production)
Coherent Scattering Lengths

\[ b = \frac{A+1}{A} a \]

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\[ n = \sqrt{1 - \frac{\lambda^2}{2\pi} \frac{Nb}{\mu B} \pm \lambda^2 \frac{2m}{(2\pi\hbar)^2}} \]
Co/Ti polarising supermirrors; K. Andersen, ILL

\[ n^2 = 1 - \frac{\lambda^2 N b}{2\pi} > 1 \]

No magnetic field

\[ \theta \]

\[ n = \begin{cases} 1 & \text{for } +p \\ >1 & \text{for } -p \end{cases} \]
Fe/Si polarising supermirrors; K. Andersen, ILL

\[ \lambda = 3 \text{Å} : l_{\text{abs}} = 70\text{cm} \]
‘remanent’ polarizing supermirrors - reversible neutron spin
magnetic anisotropy
high remanence

guide field to maintain neutron polarization $\approx 10$ G
switching of polarizer/analyzer magnetization $\Rightarrow$ short field pulse $\approx 300$ G
Crossed Supermirror
Kreuz et al./ILL+UM

![Diagram of Crossed Supermirror](image)

- **Fields in Polariser / Analyser**
  - $B_p=200$ G, $B_A=200$ G
  - $B_p=200$ G, $B_A=350$ G
  - $B_p=350$ G, $B_A=350$ G
  - $B_p=350$ G, $^3$He analyser

- **AP vs. $\lambda$ (Å)**
  - $\langle AP \rangle = 0.9901(5)$
  - $\langle AP \rangle = 0.9960(5)$
$^3$He spin filter: $n + ^3$He $\rightarrow ^1$H + $^3$H

$\sigma(J=0) = 5333 \frac{\lambda}{\lambda_0} \text{ b}; \quad \sigma(J=1) \approx 0 \quad (\lambda_0 = 1.8 \text{ Å})$
Booboo is the Cell

$^{3}$He spin filter

Cells get “milky” deposit
Evidence of Rb depolarization
Neutron Polarimetry

Opaque Analyzer: For $n_3 \sigma_a$ large enough (1-A) small enough
Can characterize Polarizer AND Flipper
\[ n + p \rightarrow d + \gamma \] (LANSCE & SNS)
Pulsed Beam Neutron Polarimetry

\[ T_{A/P} = T_0 \cosh \alpha\frac{A}{P} P_{A/P} \]
\[ T_0 = \exp(-\alpha\frac{A}{P} - \alpha_{gl} - \ldots) \]
\[ \frac{P}{A} = \tanh \alpha\frac{A}{P} P_{A/P} \]

\[ M_1 = N_0 \varepsilon_1 + B_1 \]
\[ M_2 = N_0 T_1 T_P \varepsilon_2 + B_2 \]

\[ M'_2(0)/M'_2(\text{out}) = T_0 \]
\[ M'_2(P)/M'_2(0) = \cosh \alpha_p P_p \]

(Coulter et al. NIMA 288, 463 (1989))
Pulsed Beam Neutron Polarimetry

\[ T_{A/P} = T_0 \cosh \alpha_{A/P} P_{A/P} \]
\[ T_0 = \exp(-\alpha_{A/P} - \alpha_{gl} - \ldots) \]
\[ P/A = \tanh \alpha_{A/P} P_{A/P} \]

Offsets/Backgrounds
\[ \text{Residual (10}^{-3} \text{)} \]

\[ \exp \alpha_P \]

\[ \frac{M_2(0)}{M_2(\text{out})} \]

\[ \text{wavelength (Å)} \]

\[ 0 \leq \text{wavelength (Å)} \leq 6 \]

\[ \text{Residual (10}^{-3} \text{)} \]

\[ \cosh \alpha_P P_P \rightarrow P_n = \tanh \alpha_P P_P \]

\[ \frac{M_2(P)}{M_2(0)} \]

\[ \text{wavelength (Å)} \]

\[ 0 \leq \text{wavelength (Å)} \leq 6 \]
Spin Transport and Spin Flippers

\[ \frac{d\vec{s}}{dt} = \gamma \vec{s} \times \vec{B} \]

e.g. \( \vec{B} = B_0 \hat{z} \), and \( \vec{s} = \frac{\hbar}{2} \hat{z} \) so \( \frac{d\vec{s}}{dt} = 0 \) as long as \( \frac{d\vec{B}}{dt} = 0 \)

\[ \frac{d\vec{B}}{dt} \ll \gamma B^2 \] is sufficient (adiabatic limit)

A spin in free space is hard to “depolarize” or flip

Exceptions: “diabatic”, oscillating fields

Current sheet spin flipper (also Meissner shield)
Resonant Spin Flippers

\[ \frac{d\vec{s}}{dt} = \gamma \vec{s} \times \vec{B} \]

\[ \vec{B} = B_0 \hat{z} + B_1 \cos \omega t \hat{x} \]

\[ \frac{ds_z}{dt} = \gamma B_1 \]

Spin flip: \( \gamma B_1 t = \pi \) (\( \pi \) pulse)

npdgamma spin flipper: \( t = L/v \)

\[ B_1 \]

\[ \text{time after pulse} \]
The fast adiabatic spin flipper

Beam area

Static gradient field \( \approx \) in z-direction

\[ B > B_0 \]

\[ P = +1 \]

\[ B_{\text{RF}} \text{ with fuzzy edges, set to } \omega = \omega_L = \gamma B_0 \]

\[ B_0 \]

\[ B_0 \]

\[ B_0 \]

\[ B_0 \]

\[ B < B_0 \]

\[ P = -1 \]

Spin turn in a frame \([\cdot]\) rotating with \(\omega_L\) around the z-axis:

\[ B' = B - \omega_L/\gamma = B - B_0 \]

\[ B_{x'} = B_{\text{rf}} \]
The fast adiabatic spin flipper (time domain)

\[ i\hbar \frac{d\psi}{dt} = (-\vec{\mu} \cdot \vec{B})\psi = -\left(\frac{\mu_+ B_- + \mu_- B_+}{2} + \mu_z B_z\right)\psi \]

\[ \psi = \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \]

\[ B_\pm = \frac{B_1}{2} e^{\pm i\omega_0 t} \]

\[ B_z = \beta_0 t \]

\[ i\hbar \frac{d}{dt} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = \begin{bmatrix} \mu_z \beta_0 t & \frac{B_1}{2} e^{i\omega_0 t} \\ \frac{B_1}{2} e^{-i\omega_0 t} & \mu_z \beta_0 t \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} \]

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\[ i\hbar \frac{d\psi_+}{dt} = \mu_z \beta_0 t \psi_+ + \frac{B_1}{2} e^{i\omega_0 t} \psi_- \]

\[ i\hbar \frac{d\psi_-}{dt} = \frac{B_1}{2} e^{-i\omega_0 t} \psi_+ + \mu_z \beta_0 t \psi_- \]
Rotating frame (wave) picture: \( \cos \omega t = \frac{1}{2} (e^{i\omega t} + e^{-i\omega t}) \)

\[ \Delta B = B_0 - \frac{\omega}{\gamma} \]
\[ B_x = B_1 / 2 \]

AFP: Spin follows \( B_{\text{rot}} \) adiabatically \( \left( \frac{d\theta}{dt} \ll \gamma B_1 \right) \)

Resonant flipper: \( \Delta B = 0 \)
Spin rotates around \( B_x \) (Rabi frequency \( \omega_R = \gamma \frac{B_1}{2} \))
From experiment to physics

\[
\frac{N_+ - N_-}{N_+ + N_-} = C P_n A F (1-f) + A_{\text{false}}
\]

- Spin flip efficacy
- Analyzing power
- Neutron polarization
- Background

Diagram:
- Neutrons enter with polarization \( N_0(v) \)
- Neutrons pass through polarizer and spin flipper
- Neutrons are detected by analyzer
- \( M_1 \) and \( M_2 \) represent measurement points