# Neutron Spin Echo Spectroscopy (NSE)

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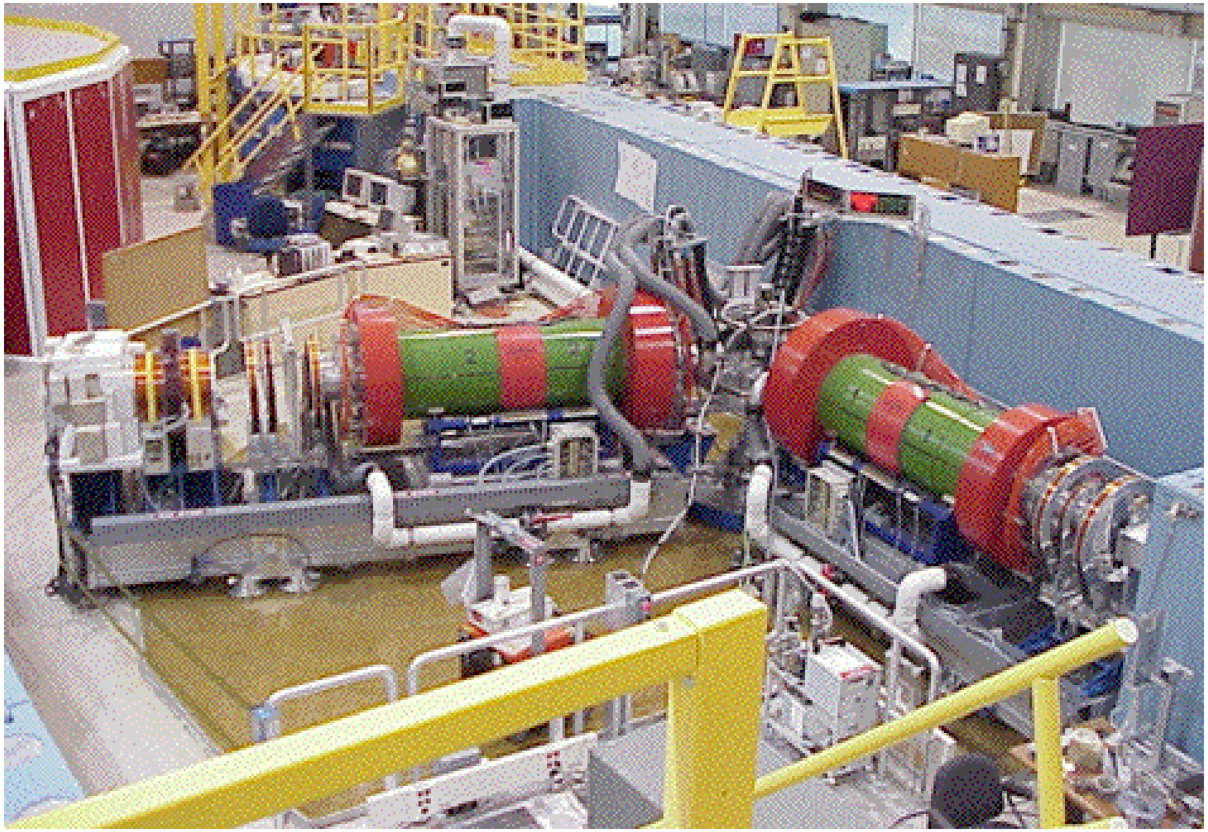

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# Why precession?

- **Goal:** $\delta E = 10^{-5} - 10^{-2}$ **meV** (very small !!!)

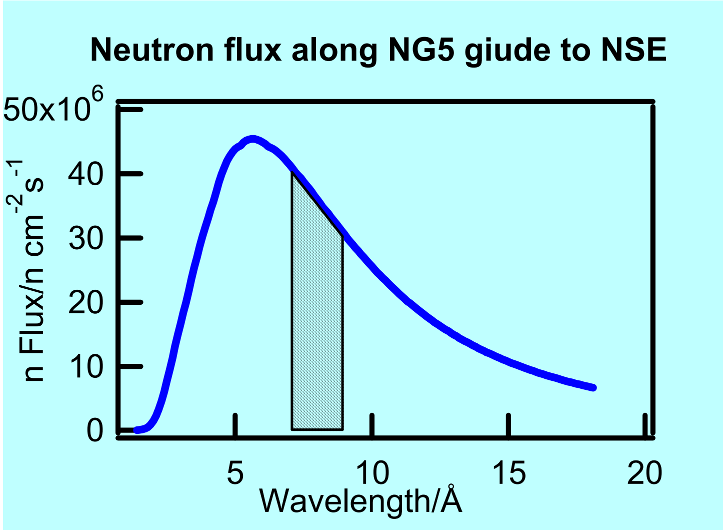
- We need low energy neutrons. Cold neutrons: $\lambda = 5 - 12$ Å, $E = 0.5 - 3.3$ meV

- **The problem:** neutron beam wavelength spread $\Delta\lambda/\lambda = 5 - 20\%$, $\Delta E/E = 10 - 40\%$, $\Delta E =$ **0.05 – 0.2 meV**

$\Delta E =$ **0.05 – 0.2 meV >>** $\delta E = 10^{-5} - 10^{-2}$ **meV**

- In fact, to measure neutron energy we need to measure the neutron velocity:

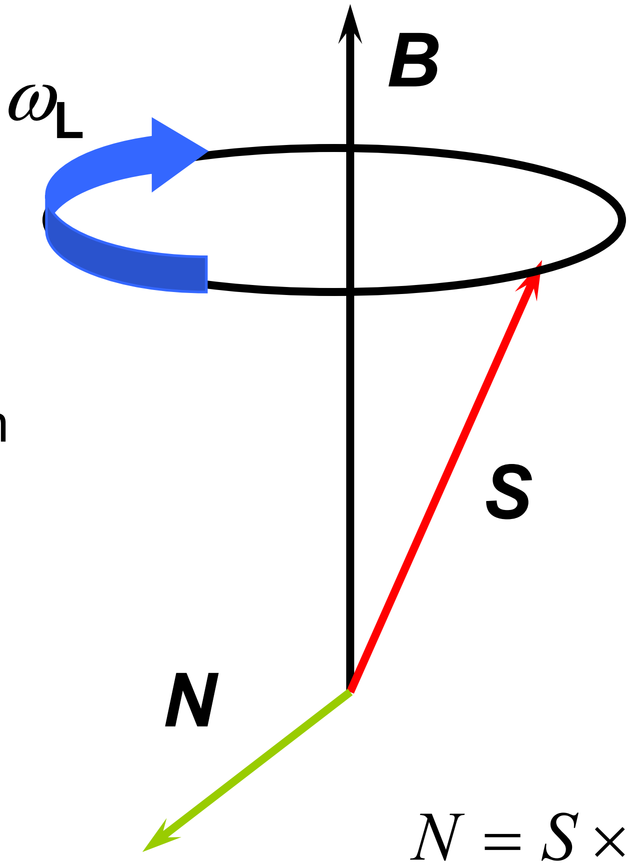
$E = mV^2/2 \quad \rightarrow \quad V = l/t \quad \rightarrow \quad t?$



- **The solution**: We need neutron precession in magnetic field. We are going to attach “internal” clock for each neutron. Thus, we can observe very small velocity changes of a neutron beam, regardless of the velocity spread

# Neutrons in magnetic fields: Precession

Mass, $m_n$ = 1.675×10$^{-27}$ kg
Spin, $S$ = 1/2 [in units of $h/(2\pi)$]
Nuclear $g$ number, $g_n = \mu_n/\mu_N$ = -1.9130
where ($\mu_N$ = 5.0508×10$^{-27}$ J/T)
Gyromagnetic ratio $g = \mu_n/[S\times h/(2\pi)]$ =
1.832×10$^{8}$ s$^{-1}$T$^{-1}$ (29.164 MHz T$^{-1}$)



- The neutron will experience a torque from a magnetic field $B$ perpendicular to its spin direction.

- Precession with the Larmor frequency:

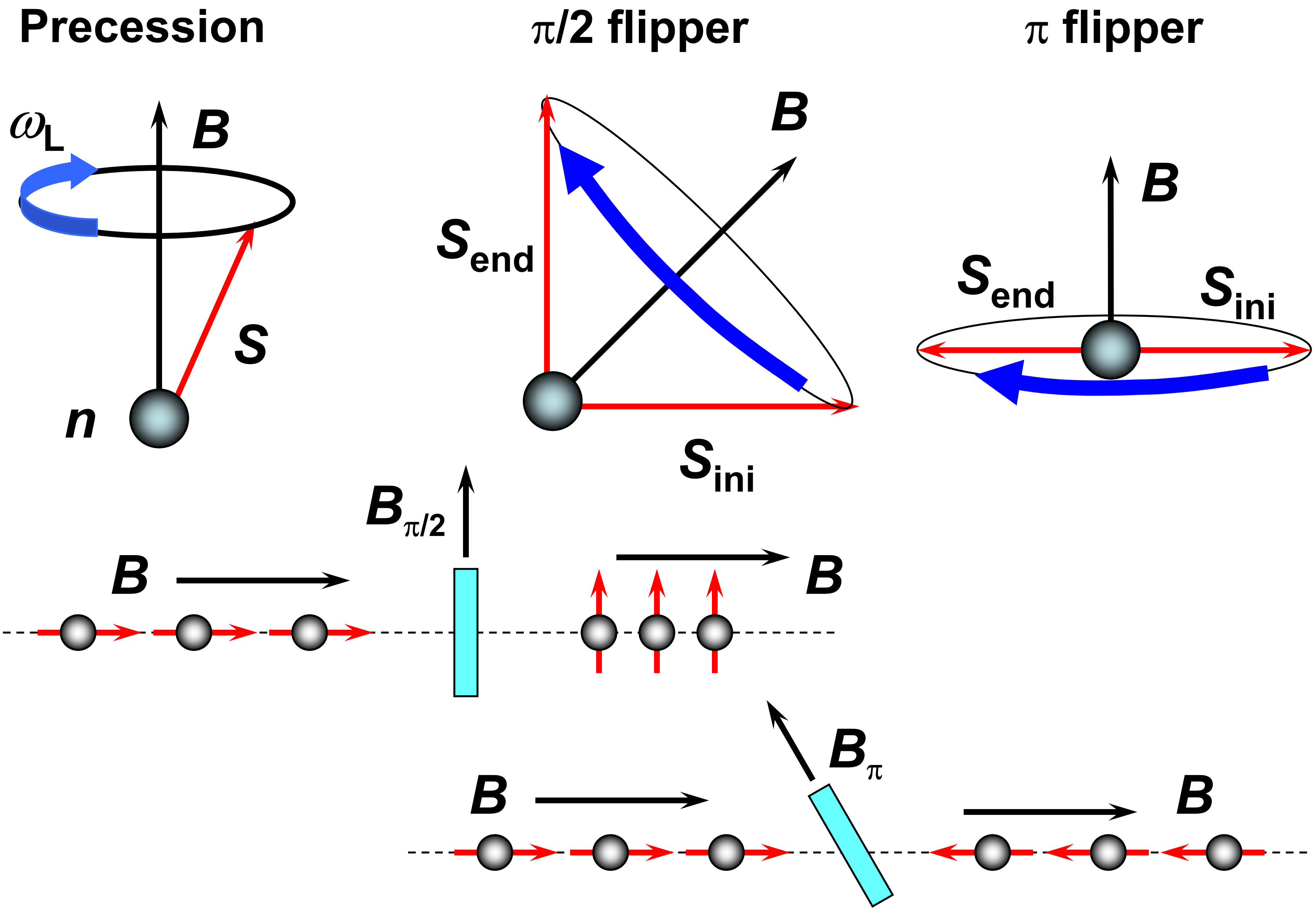
$$\omega_L = gB$$

- The precession rate is predetermined by the strength of the field only.

$$N = S \times B$$

$$\frac{dS}{dt} = \gamma S \times B = S \times \omega_L$$

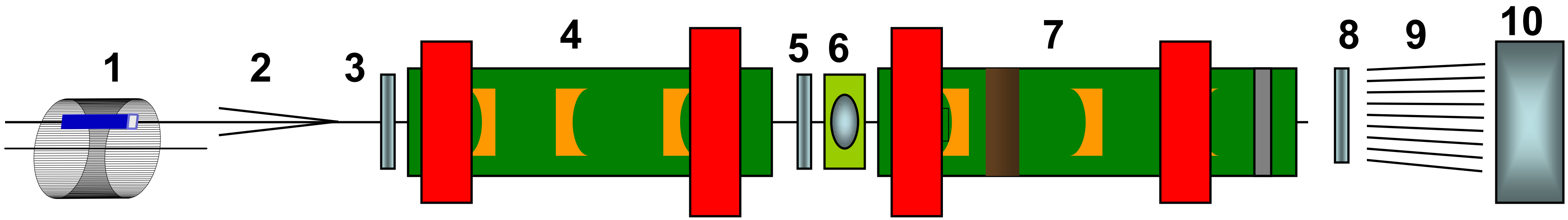
# Spin flippers


Precession

π/2 flipper

π flipper

# NSE Spectrometer schematic



**1. Velocity selector**
(selects neutrons with certain $\lambda_0$)

**2. Polarizer**
(polarizing supermirrors)

**3. $\pi/2$ flipper**
(starts Larmor precession)

**4. First main solenoid**
(field integral ~0.5 T.m)

**5. $\pi$ flipper**
(provides phase inversion)

**6. Sample**

**7. Second main solenoid**
(phase and correction coils)

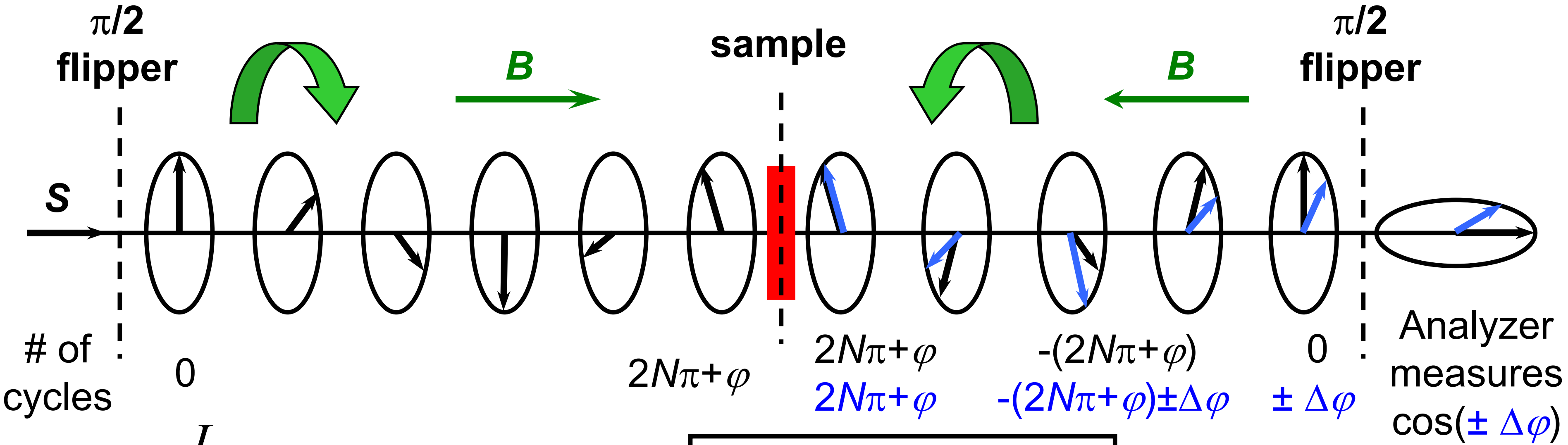
**8. $\pi/2$ flipper**
(stops Larmor precession)

**9. Polarization analyzer**
(radial array of polarizing supermirrors)

**10. Area detector**
($20\times20$ cm$^2$)

# Monochromatic beam

- elastic scattering
- inelastic scattering



π/2 flipper

*B*

sample

*B*

π/2 flipper

*S*

Analyzer

\# of cycles

0

2*N*π+*φ*

2*N*π+*φ*
2*N*π+*φ*

-(2*N*π+*φ*)
-(2*N*π+*φ*)±Δ*φ*

0
± Δ*φ*

Analyzer measures cos(± Δ*φ*)

$$\varphi = gB\frac{L}{v}$$

$$N(\lambda) = \frac{1}{2\pi}\int \frac{4\pi g\mu_N Bm\lambda}{h^2}dl =$$

$$= \frac{2g\mu_N m\lambda}{h^2}\int Bdl =$$

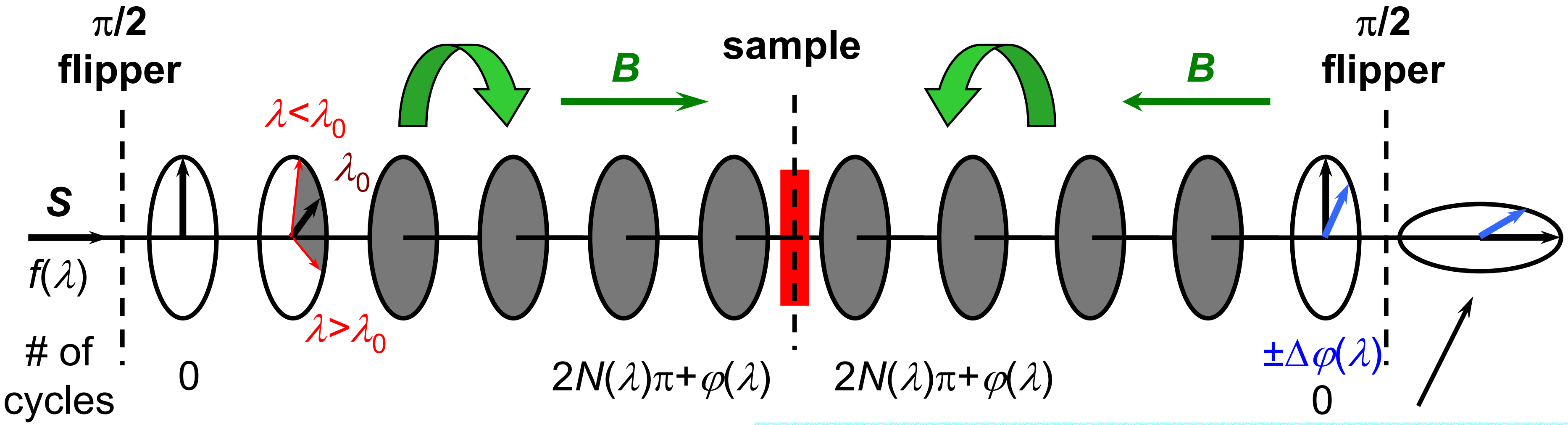
$$= 7370 \times \mathrm{J}[\mathrm{T.m}] \times \lambda[\text{Å}]$$

*J* – field integral
At NCNR:
$J_{\max} = 0.5$ T.m
$N(\lambda = 8\text{Å}) \sim 3\times10^5$

$$\frac{\Delta v}{v} \approx \frac{1}{N} \approx 10^{-5}\ !$$

$$\Delta\varphi = gBL\left(\frac{1}{v} - \frac{1}{v'}\right) = \frac{gBL\Delta v}{v^2}$$

# Polychromatic beam



The analyzer projects out the spin component parallel to the beam, cos(Δφ(λ)):

Define $N_0 \equiv N(\lambda_0)$; then $N(\lambda) = N_0 \dfrac{\lambda}{\lambda_0}$

and $\varphi'(\lambda) = N_0 \dfrac{\delta\lambda}{\lambda_0} + \Delta N_0 \dfrac{\lambda}{\lambda_0} + \Delta N_0 \dfrac{\delta\lambda}{\lambda_0}$.

$$\cos\left[2\pi(\mathrm{N}_0\delta\lambda + \Delta N_0\lambda)/\lambda_0\right] =$$

$$= \cos\left(2\pi\mathrm{N}_0\frac{\delta\lambda}{\lambda_0}\right)\cos\left(2\pi\Delta N_0\frac{\lambda}{\lambda_0}\right) + \text{2nd order terms}$$

Energy change

Asymmetry between coil field integrals

Neglect 2nd order terms for small asymmetries or quasielastic scattering.

Neglected

# Intensity at the detector

For a single wavelength:

$$\cos\left(2\pi \mathrm{N}_0 \frac{\delta\lambda}{\lambda_0}\right)\cos\left(2\pi \Delta N_0 \frac{\lambda}{\lambda_0}\right)$$

For wavelength distribution, *f*($\lambda$), with mean wavelength, $\lambda_0$:

$$\langle P\rangle = \int_0^{\infty} f(\lambda)\cos\left(2\pi \Delta N_0 \frac{\lambda}{\lambda_0}\right)\left[\int_{-\infty}^{\infty} S(\mathbf{Q},\omega)\cos(\omega t(\lambda))\,d\omega\right]d\lambda$$

$$\text{where } t \equiv \frac{N_0 m \lambda^3}{h\lambda_0} \quad \text{since } \delta\lambda = \frac{m\lambda^3}{2\pi h}\omega$$

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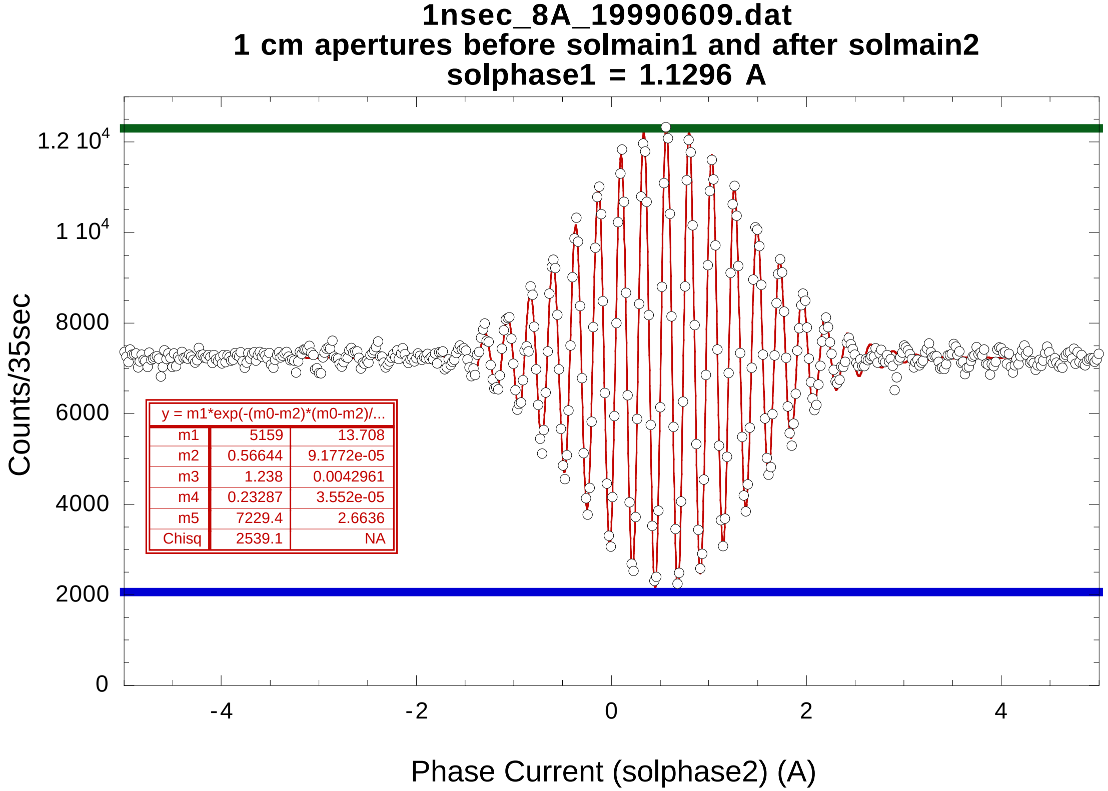
At $t = 0$:

$$\left[\int_{-\infty}^{\infty} S(\mathbf{Q},\omega)\cos(\omega t(\lambda))\,d\omega\right] \Rightarrow S(\mathbf{Q})$$

$$\langle P\rangle = \int_0^{\infty} f(\lambda)\cos\left(2\pi \Delta N_0 \frac{\lambda}{\lambda_0}\right)d\lambda$$

At small $\mathrm{N}_0$ vary $\Delta\mathrm{N}_0$:

- Oscillations give $\lambda_0$
- Envelope gives *f*($\lambda$)



**1nsec_8A_19990609.dat**
**1 cm apertures before solmain1 and after solmain2**
**solphase1 = 1.1296 A**

| y = m1*exp(-(m0-m2)*(m0-m2)/... | | |
|---|---|---|
| m1 | 5159 | 13.708 |
| m2 | 0.56644 | 9.1772e-05 |
| m3 | 1.238 | 0.0042961 |
| m4 | 0.23287 | 3.552e-05 |
| m5 | 7229.4 | 2.6636 |
| Chisq | 2539.1 | NA |

# How to deal with the resolution?

$$\langle P\rangle = \int_0^\infty f(\lambda)\cos\left(2\pi\Delta N_0 \frac{\lambda}{\lambda_0}\right)\left[\int_{-\infty}^{\infty} S(\mathbf{Q},\omega)\cos(\omega t(\lambda))d\omega\right]d\lambda$$

$$J(\mathbf{Q},\omega) = S(\mathbf{Q},\omega) \otimes R(\mathbf{Q},\omega)$$

In the energy domain, the energy resolution of the spectrometer is convoluted with the scattering properties of the sample

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Convert to the time domain :

$$\left[\int_{-\infty}^{\infty} S(\mathbf{Q},\omega)\cos(\omega t(\lambda))d\omega\right] = I(\mathbf{Q},t(\lambda))$$

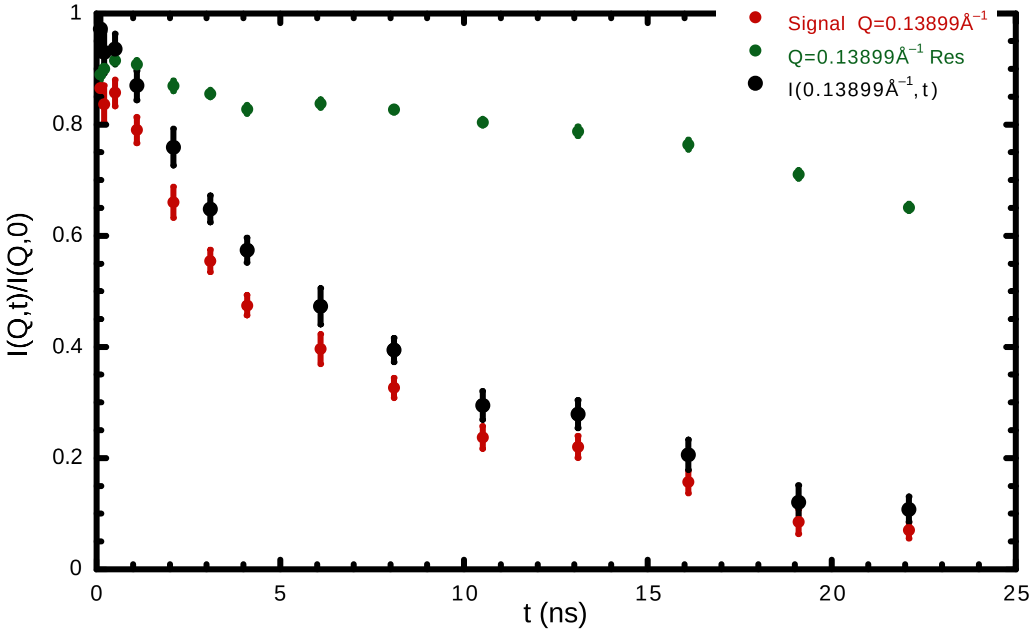
At the echo point, $\Delta N_0 = 0$,

$$\langle P\rangle = \int_0^\infty f(\lambda) I(\mathbf{Q},t(\lambda))d\lambda$$

$$J(\mathbf{Q},t) = I(\mathbf{Q},t)\cdot R(\mathbf{Q},t)$$

$$I(\mathbf{Q},t) = \frac{J(\mathbf{Q},t)}{R(\mathbf{Q},t)}$$

In the time domain the deconvolution is simply a division.



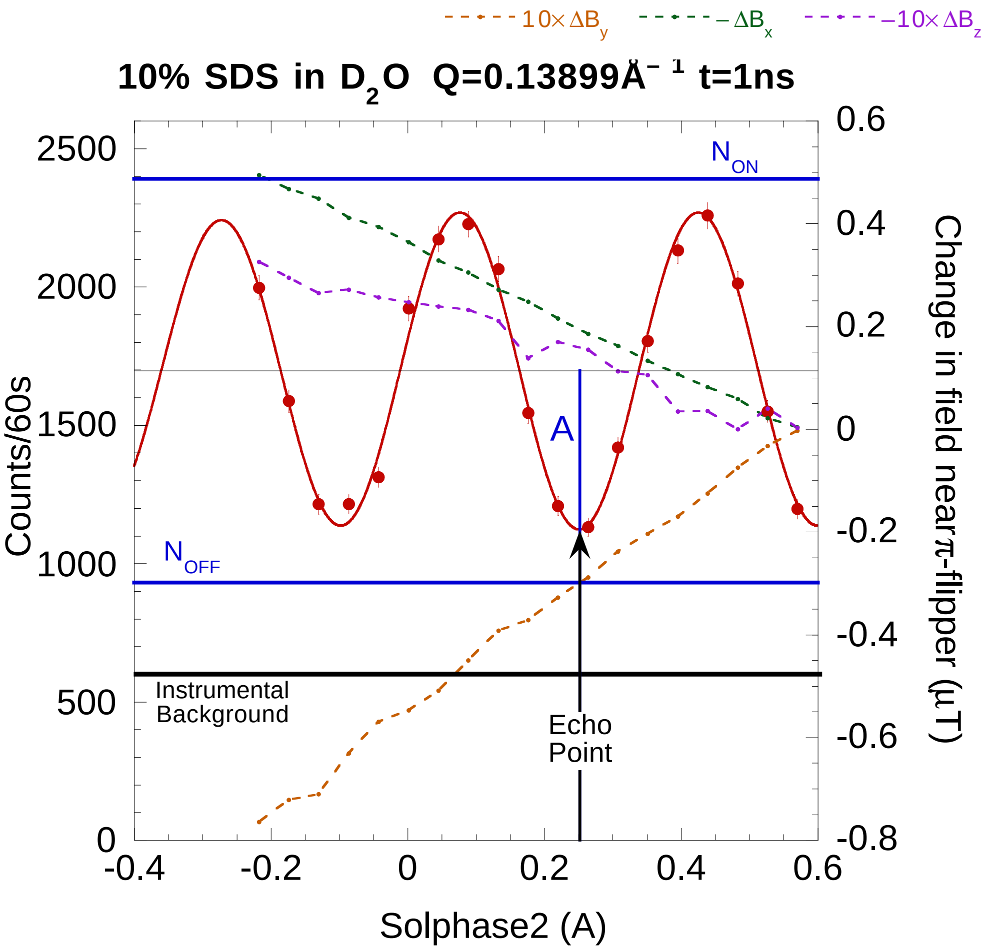
# Measuring *I*(*Q*,*t*)

- The difference between the flipper ON and flipper OFF data gives $I(Q,0)$

- The echo is fit to a gaussian-damped cosine.

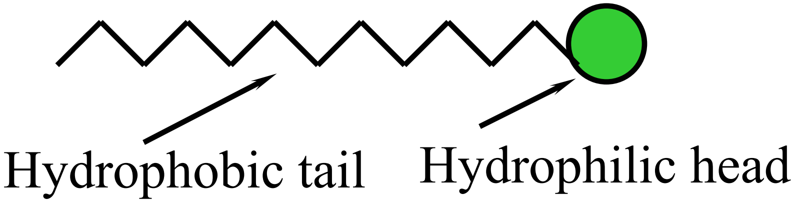
Signal before resolution correction is

$$\frac{2A}{N_{ON} - N_{OFF}}$$

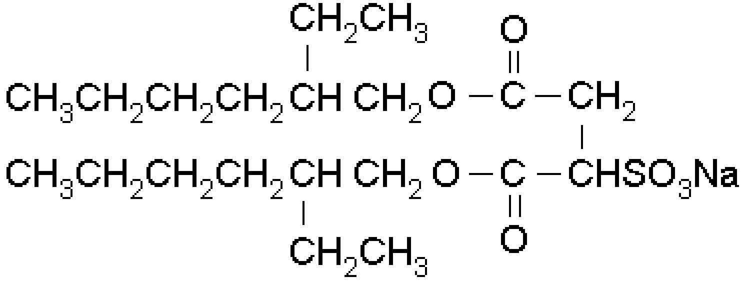


# Experimental system

**Surfactant molecule**



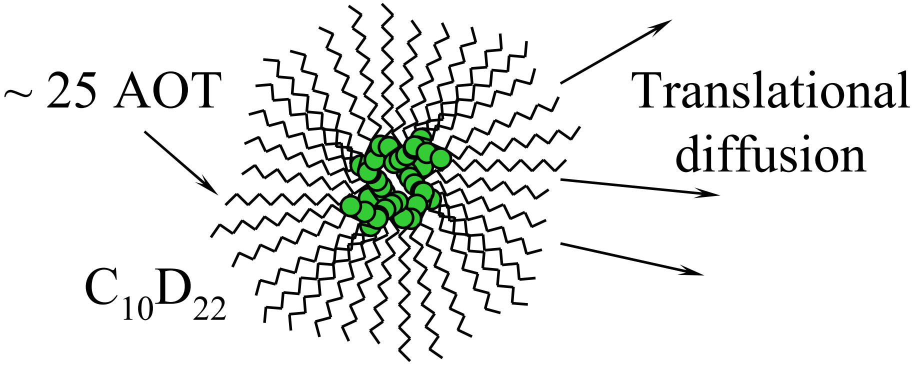
**AOT**



## Experiment I

**Diffusion** of AOT micelles in $C_{10}D_{22}$
(5.4 % vol. fraction)

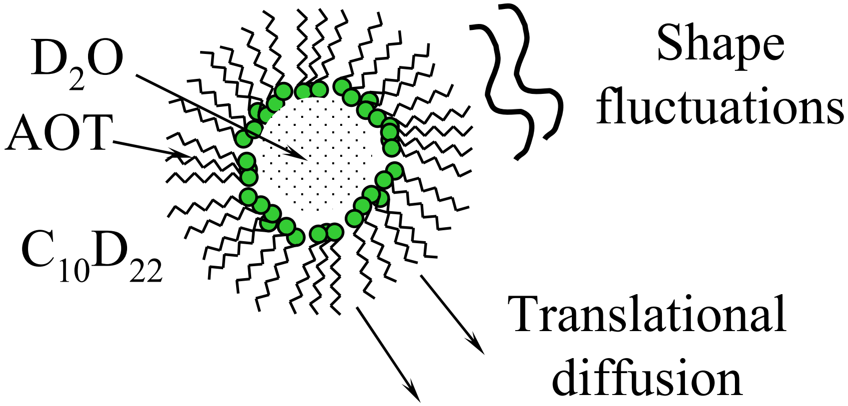
**Inverse spherical micelle**



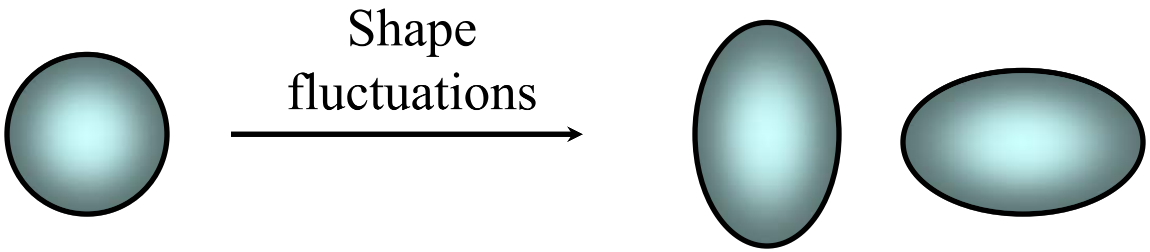
## Experiment II

**Shape fluctuations** in
$AOT/D_2O/C_{10}D_{22}$ microemulsion
(5.4/4.6/90 % vol. fraction)

**Inverse microemulsion droplet**

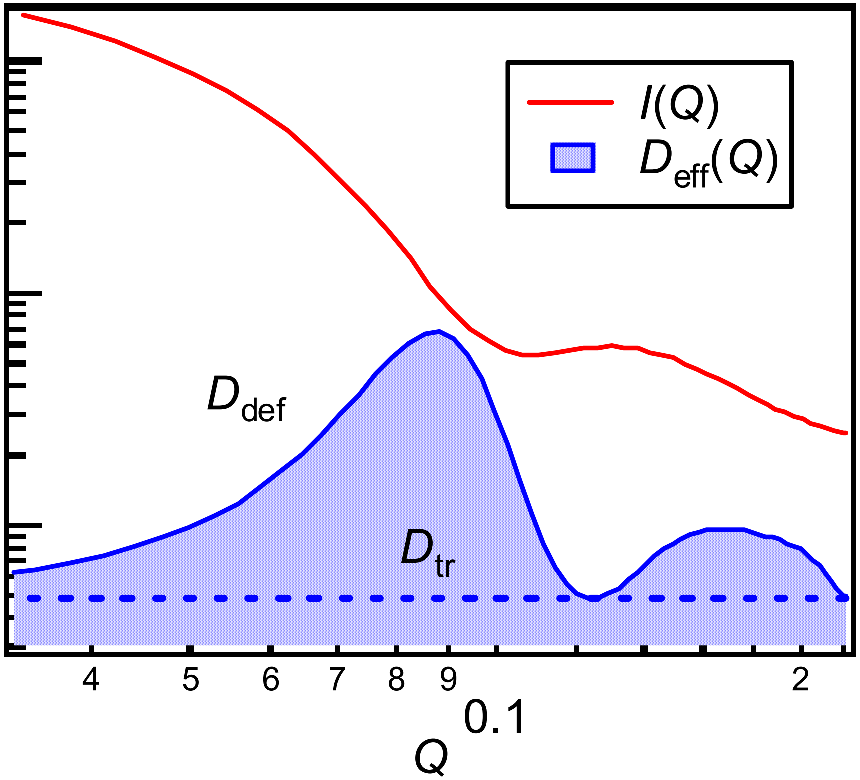


# Data analysis



Shape fluctuations

$$E_{bend} = \frac{k}{2}\int dS\left(\frac{1}{R_1}+\frac{1}{R_2}-\frac{2}{R_s}\right)+\bar{k}\int dS\,\frac{1}{R_1R_2}$$



Expansion of $r$ in spherical harmonics with amplitude $a$:

$$r(\Omega) = r_0\left(1+\sum_{l,m} a_{lm}\,\mathrm{Y}_{lm}(\Omega)\right)$$

Frequency of oscillations of a droplet:

$$\lambda_2 = \frac{k}{\eta R_0^3}\left[4\frac{R_0}{R_s}-3\frac{\bar{k}}{k}-\frac{3k_BT}{4\pi k}f(\phi)\right]\frac{24\eta}{23\eta'+32\eta}$$

# Summary of data analysis

**Experiment I**

AOT micelles in $C_{10}D_{22}$ →

$$\frac{I(Q,t)}{I(Q,0)} = \exp\left[-D_{eff}Q^2t\right]$$

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**Experiment II**

AOT/$D_2O$/$C_{10}D_{22}$ microemulsion →

$$\frac{I(Q,t)}{I(Q,0)} = \exp\left[-D_{eff}(Q)Q^2t\right]$$

$$D_{eff}(Q) = D_{tr} + D_{def}(Q)$$

$$D_{eff}(Q) = D_{tr} + \frac{5\lambda_2 f_2(QR_0)\left\langle |a_2|^2 \right\rangle}{Q^2\left[4\pi[j_0(QR_0)]^2 + 5f_2(QR_0)\left\langle |a_2|^2 \right\rangle\right]}$$

**Goal:** Bending modulus of elasticity

$$k = \frac{1}{48}\left[\frac{k_BT}{\pi p^2} + \lambda_2\eta R_0^3\,\frac{23\eta' + 32\eta}{3\eta}\right]$$

$$f_2(QR_0) = 5\left[4j_2(QR_0) - QR_0 j_3(QR_0)\right]^2$$

$\lambda_2$ – the damping frequency – **frequency of deformation**

$<|a|^2>$ – mean square displacement of the 2-nd harmonic – **amplitude of deformation**

$p^2$ – size polydispersity, measurable by SANS or DLS