



A Fast and Self-consistent Approach for Multi-cycle Equilibrium Core Studies Using Monte Carlo Models

Zeyun Wu and Robert Williams

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Introduction

- ▶ Monte Carlo (MC) approach became inefficient for nuclear applications requiring number of repeated transport calculations.
- ▶ Multi-cycle equilibrium core development, the very first task encountered by reactor physicists, is one of the time-consuming problems of this type with MC approaches.
- ▶ An efficient and self-consistent approach, PRELIM approach, is developed to reduce the computational overhead on MC models when applied to feasibility studies in multi-cycle core design and analysis.
- ▶ The primary goal of this work is to provide a simplified but rigorous shortcut to quickly generate an equilibrium core configurations for routine reactor calculations using MC models, particularly for feasibility or optimized studies on a new reactor design.

Overview of the methodology

- ▶ The method is built based on simple reactor physics theory and is easily implemented, whereas it can produce numerical results with an accuracy as the same level as higher order calculations.
- ▶ The approach starts with an initial estimate of the fuel material for an equilibrium core at the end of cycle (**EOC**) given basic core design parameters.
- ▶ The initial values of the core inventories are adjusted in subsequent iterations using reaction rates of interest and thermal fluxes calculated by MC simulation.
- ▶ At the end, fuel contents in the startup (**SU**) and beginning of cycle (**BOC**) cores can also be produced by using the results in EOC, fuel cycle length and core configuration.
- ▶ The approach is not intended for reactor safety analysis, but rather aimed for providing a quick approach for neutronics feasibility studies.

Fundamentals for the PRELIM Approach¹

- ▶ The goal of the PRELIM approach is to generate fuel inventories of these selected materials for equilibrium cores at SU, BOC, and EOC respectively. The overwhelming fissionable isotopes such as U-235, U-238, Pu-239 are explicitly treated as well as the most significant fission product poisons such as Xe-135 and Sm-149.
- ▶ I-135/Xe-135 solution

$$N_I(t) = \frac{\gamma_I \Sigma_f \phi_0}{\lambda_I} (1 - e^{-\lambda_I t}), \quad N_X(t) = \frac{(\gamma_X + \gamma_I) \Sigma_f \phi_0}{\lambda_X + \sigma_a^X \phi_0} \left[1 - e^{-(\lambda_X + \sigma_a^X \phi_0)t} \right] + \frac{\gamma_I \Sigma_f \phi_0}{\lambda_X - \lambda_I + \sigma_a^X \phi_0} \left[e^{-(\lambda_X + \sigma_a^X \phi_0)t} - e^{-\lambda_I t} \right]$$

- ▶ Pm-149/Sm-149 solution

$$N_P(t) = \frac{\gamma_P \Sigma_f \phi_0}{\lambda_P} (1 - e^{-\lambda_P t}), \quad N_S(t) = S^{SU} e^{-\sigma_a^S \phi_0 t} + \gamma_P \Sigma_f \phi_0 \left[\frac{1 - e^{-\sigma_a^S \phi_0 t}}{\sigma_a^S \phi_0} + \frac{e^{-\sigma_a^S \phi_0 t} - e^{-\lambda_P t}}{\sigma_a^S \phi_0 - \lambda_P} \right]$$

- ▶ U-238/Pu-239 solution

$$N_{39}(t) = \frac{\sigma_\gamma^{28} N_{28} \phi_0}{\lambda_{39}} (1 - e^{-\lambda_{39} t}), \quad N_{49}(t) = N_{49}^{SU} e^{-(\sigma_\gamma^{49} + \sigma_f^{49}) \phi_0 t} + \sigma_\gamma^{28} N_{28} \phi_0 \left[\frac{1 - e^{-(\sigma_\gamma^{49} + \sigma_f^{49}) \phi_0 t}}{(\sigma_\gamma^{49} + \sigma_f^{49}) \phi_0} + \frac{e^{-(\sigma_\gamma^{49} + \sigma_f^{49}) \phi_0 t} - e^{-\lambda_{39} t}}{(\sigma_\gamma^{49} + \sigma_f^{49}) \phi_0 - \lambda_{39}} \right]$$

[1]. J. J. Duderstadt and L. J. Hamilton, Nuclear Reactor Analysis, John Wiley & Sons, (1976)

The procedure of the PRELIM approach (1)

▶ Step I: Initial Values of EOC Model

- The consumed fissile material U-235 at EOC can be estimated based on the fuel cycle length and power rate of the reactor.
- The buildup of Pu-239 and depletion of U-238 at EOC can also be estimated accordingly.

- Saturated Xe-135 concentration at EOC

$$N_X^\infty = \frac{(\gamma_X + \gamma_I)\Sigma_f}{\sigma_a^X} \approx \frac{(\gamma_X^{25} + \gamma_I^{25})\sigma_f^{25}N_{25} + (\gamma_X^{49} + \gamma_I^{49})\sigma_f^{49}N_{49}}{\sigma_a^X}$$

- Saturated Sm-149 concentration at EOC

$$N_S^\infty = \frac{\gamma_P\Sigma_f}{\sigma_a^S} \approx \frac{\gamma_P^{25}\sigma_f^{25}N_{25} + \gamma_P^{49}\sigma_f^{49}N_{49}}{\sigma_a^S}$$

- Other fission product poisons

$$\sigma_a^B N_B = \sigma_g (N_{25}^0 - N_{25}) \frac{\sigma_f^{25}}{\sigma_a^{25}}, \text{ where } \sigma_g = 50 \text{ barn.}$$

- An optional “filler” material, Bi-209, is used in the approach to account for the rest of burned fuel mass to preserve the same fuel density as fresh fuel.

The procedure of the PRELIM approach (2)

▶ Step II: Iterations on EOC Model

- Update Xe-135 concentration at EOC

$$N'_X = \frac{(\gamma_X + \gamma_I) R_f}{\lambda_X + R_X / N_X} = \frac{(\gamma_X^{25} + \gamma_I^{25}) R_f^{25} + (\gamma_X^{49} + \gamma_I^{49}) R_f^{49}}{\lambda_X + R_X / N_X}$$

- Update Sm-149 concentration at EOC

$$N'_S = N_S \frac{\gamma_P R_f}{R_S} = N_S \frac{\gamma_P^{25} R_f^{25} + \gamma_P^{49} R_f^{49}}{R_S}$$

- Update B-10 concentration at EOC for minor FP poisons

$$N'_B = \frac{(\sigma_g \cdot 10^{-24})(N_{25}^0 - N_{25}) \varphi_{th} \sigma_f^{25}}{R_B} \frac{\sigma_f^{25}}{\sigma_a^{25}} N_B$$

- Update consumed U-235 mass at EOC

$$m_{25} = \frac{N_{tot}^{25}}{N_A} M_{25} = \frac{R_{25} V T_c}{N_A} M_{25}$$

where V is the fuel volume, T_c is the fuel cycle length, M_{25} is the atomic mass of U-235, N_A is the Avogadro constant.

- Update Pu-239 concentration at EOC

$$N_{49} = N_{49}^{SU} e^{-\lambda_{49} T_c} + R_{28} \left[\frac{1 - e^{-\lambda_{49} T_c}}{\lambda_{49}} + \frac{e^{-\lambda_{49} T_c} - e^{-\lambda_{39} T_c}}{\lambda_{49} - \lambda_{39}} \right]$$

Note: the highlighted quantities in the equations are obtained from MC tallies from the pervious iteration.

The procedure of the PRELIM approach (3)

▶ Step III: Startup (SU) Core Model

- For the SU core, there is no Xe in any fuel, and there is no Sm, Pu, B nor Bi in fresh fuels. If the period between cycles is a week or so, we can assume Pm-149 has all converted to Sm-149, and Np-239 has all converted to Pu-239.

- Sm-149 concentration at SU

$$N_P^\infty = N_P^{EOC} = \frac{\gamma_P \Sigma_f \phi_0}{\lambda_P} = \frac{\gamma_P R_f}{\lambda_P} = \frac{\gamma_P^{25} R_f^{25} + \gamma_P^{49} R_f^{49}}{\lambda_P}, \quad N_S^{SU} = N_S^{EOC} + N_P^{EOC}$$

- Pu-239 concentration at SU

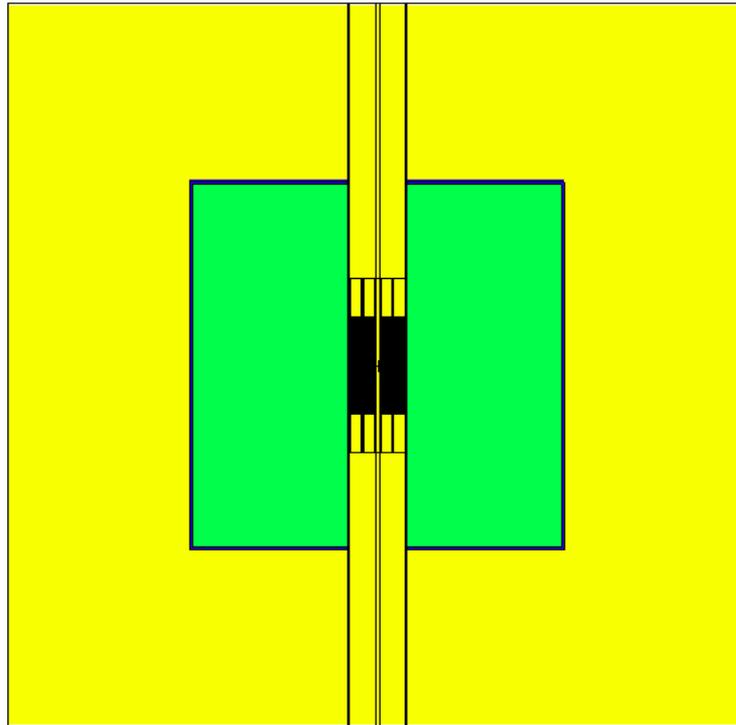
$$N_{39}^\infty = N_{39}^{EOC} = \frac{\sigma_\gamma^{28} N_{28} \phi_0}{\lambda_{39}} = \frac{R_{28}}{\lambda_{39}}, \quad N_{49}^{SU} = N_{49}^{EOC} + N_{39}^{EOC}$$

- The concentration of all other main constituents at SU can be assumed to be unchanged to the one in EOC model in the previous burnt cycle.

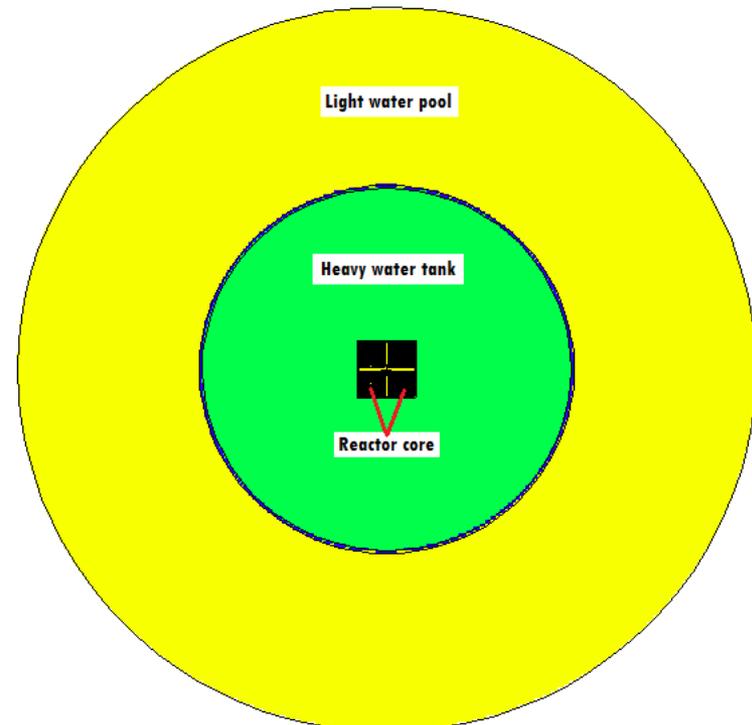
▶ Step IV: BOC Core Model

- The BOC core here is defined as the SU core with addition of equilibrium concentration of Xe-135 in the fuel. The mass concentration of Xe-135 at EOC can be used as the one for BOC.
- For simplicity, the Sm-149 and Pu-239 concentrations at BOC are obtained as the average of the SU and EOC values for a specific fuel material.

Numerical Example – OPAL Like Core



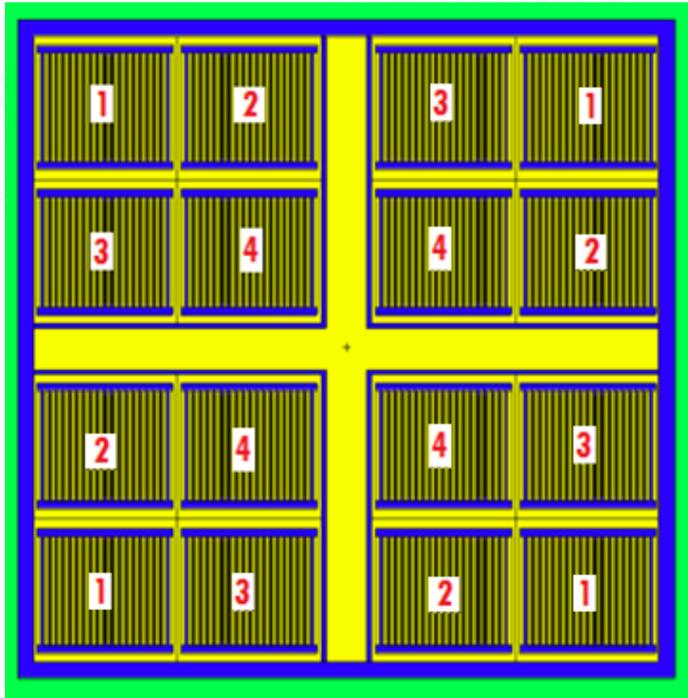
(a) X-Z view



(b) X-Y view

A Schematic view of cutaway side-plane (left) and mid-plane (right) of the reactor.

Fuel Element Layout in the Core

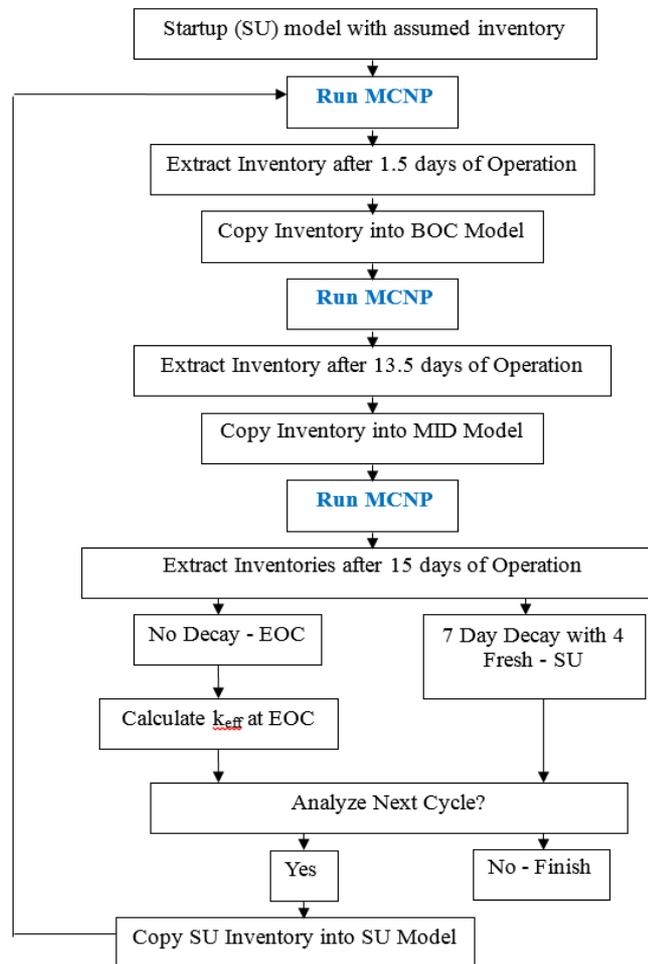


The core is configured in a symmetric 4 x 4 layout geometry. The loading and shuffling scheme for fuel elements in the core is shown as red numbers in the figure, in which the number 1 stands for the fresh fuel at SU, and number 4 stands for the discard fuel at EOC.

Core design Parameter by MCNP6

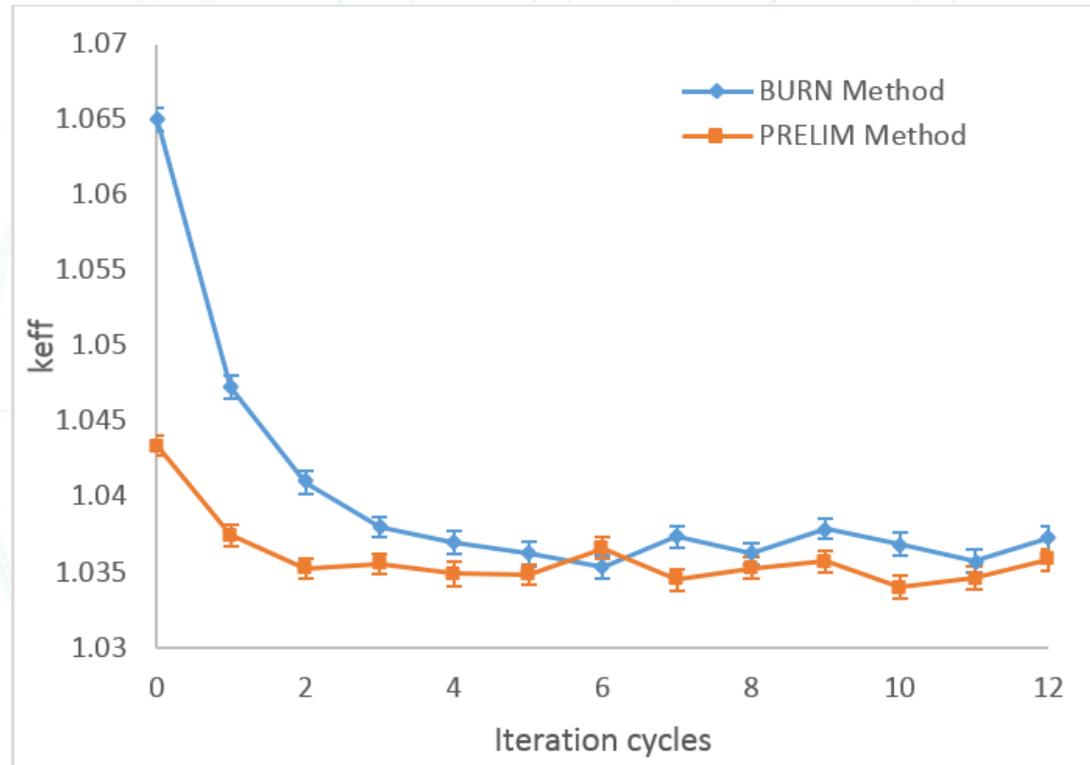
Parameter	Data
Power rate (MW _{th})	20
Reactor type	Tank in a pool
Fuel cycle length (days)	30
Days between cycles	7
Fuel cycle batches	4
Fuel element (FE) layout	4 x 4
Fuel type	MTR
Number of fuel plates per FE	17
Fuel material	U ₃ Si ₂
Fuel enrichment (%)	19.75
Fuel density (g/cc)	6.52
Fuel volume per FE (cc)	6.52
Reactor coolant/moderator	Light water
Reactor reflector	Heavy water
Biological and thermal shielding	Light water pool

MCNP6 BURN Approach¹



[1]. A. Hanson and D. Diamond, "A Neutronics Methodology for the NIST Research Reactor Based on MCNPX," in the 19th International Conference on Nuclear Engineering (ICONE-19), Chiba, Japan, May 16-19 (2011).

EOC k_{eff} Changes along iterative cycles in both approaches



The k_{eff} yielded from the two approaches have significant difference in the first few iterations, this is mainly because the BURN approach used all fresh fuel at the starting point. After about 4 iterations, the two k_{eff} curves both asymptotically converge to roughly the same value at EOC.

Comparison of k_{eff} at SU, BOC and EOC

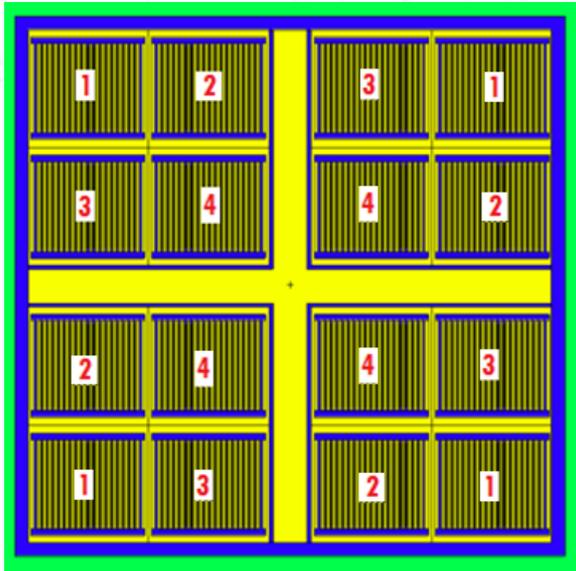
Stage	BURN	PRELIM	Deviation
SU	1.11535 ± 0.00084	1.11442 ± 0.00090	0.00093
BOC	1.07756 ± 0.00075	1.07521 ± 0.00083	0.00235
EOC	1.03729 ± 0.00083	1.03583 ± 0.00076	0.00146

The **computational time** demanded by the BURN approach is far more than the time needed in the PRELIM approach. In this problem, with the same number of starting particles provided for the *kcode* calculation in MCNP (200 active cycles with 5000 particles per cycle), the average computation time is about **200 minutes** per iteration cycle in the BURN approach, while the proposed PRELIM approach only takes about **6 minutes** to complete one iteration cycle calculation. Note the time compared here is the wall clock time on executing MCNP6 in a single desktop with 8 processor CPUs at 3.40 GHz.

Prediction of mass fractions of some key isotopes in burnt fuels at EOC

		U-235	Pu-239	Xe-135	Sm-149
Once burnt fuel	BURN	1.27E-01	1.09E-03	8.63E-07	6.95E-06
	PRELIM	1.27E-01	1.19E-03	8.78E-07	7.31E-06
	Difference (%)	-0.61	8.51	1.78	5.17
Twice burnt fuel	BURN	1.10E-01	2.05E-03	7.66E-07	6.46E-06
	PRELIM	1.08E-01	2.19E-03	7.81E-07	6.54E-06
	Difference (%)	-1.58	7.10	1.86	1.22
Third burnt fuel	BURN	9.40E-02	2.74E-03	6.69E-07	5.94E-06
	PRELIM	9.14E-02	2.87E-03	6.72E-07	5.70E-06
	Difference (%)	-2.75	4.75	0.46	-4.06
Fourth burnt fuel	BURN	7.85E-02	3.19E-03	5.75E-07	5.36E-06
	PRELIM	7.58E-02	3.27E-03	5.62E-07	4.81E-06
	Difference (%)	-3.39	2.63	-2.32	-10.34

Power distribution at EOC



1.04	0.97	1.04	1.04
1.05	0.97	1.04	1.05
-0.81	0.53	-0.24	-0.56
1.03	0.95	0.95	0.97
1.04	0.94	0.94	0.97
-0.38	0.92	1.06	0.36
0.97	0.95	0.95	1.03
0.97	0.94	0.94	1.04
0.09	1.10	1.02	-0.31
1.04	1.04	0.97	1.04
1.05	1.04	0.97	1.05
-1.02	-0.59	0.08	-0.85

BURN
PRELIM
Diff (%)

The power factors of fuel elements predicted by the two approaches. A group of three values are shown for each fuel element: The first value is the power factor predicted by the **BURN** approach, the second value is the one predicted by the **PRELIM** approach, and the last one gives the relative difference between these two. Colors in the figure indicates the magnitude of the normalized power of the FE.

Summary

- ▶ A fast and self-consistent approach, PRELIM approach, is presented to quickly achieve multi-cycle equilibrium core for feasibility studies in new reactor design using MC models.
- ▶ The computational time required to produce an equilibrium core, as shown in the example problem, is significantly reduced comparing to the approach introduced by the latest MCNP code.
- ▶ The primary advantage of the approach is that it enables conceptual core design calculations in a repeated manner with sufficient accuracy on key design performance parameters such as k_{eff} , flux and power distribution, etc.
- ▶ The approach is desirable in core feasibility studies but once a conceptual design is chosen, more rigorous methods are needed for fuel depletion analyses and the reactor safety analysis.