Focusing cold neutrons with multiple biconcave lenses for small-angle neutron scattering


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The focusing of a cold neutron beam by multiple biconcave lenses has recently been proposed as a practical means of extending the lower limit of \(Q\) in conventional, long flight-path small-angle neutron scattering (SANS) instruments. To test the feasibility of this approach, we have carried out extensive measurements on one of the 30 m SANS instruments at NIST of the focusing characteristics of a set of 28 biconcave MgF\(_2\) lenses. The focused beam profile has been measured over several orders of magnitude using high resolution neutron auto-radiography. The focusing lens configuration outperforms the pinhole collimation at \(Q_{\text{min}}\) lower than 0.004 Å\(^{-1}\). At \(Q_{\text{min}} = 0.001\,\text{Å}^{-1}\), the intensity gain of the lens configuration over the pinhole collimation is greater than one order of magnitude.

**Keywords:** small-angle neutron scattering, neutron focusing, refractive lens.

1. Introduction

The focusing of a cold neutron beam by multiple biconcave lenses has recently been demonstrated (Eskildsen et al., 1998) and proposed as a practical means of improving the lower limit of \(Q\) in conventional SANS instruments which use circular apertures separated by distances of several meters (pinhole collimation) to collimate the SANS instruments which use circular apertures separated by pinhole collimation conditions (Mildner & Carpenter, 1984). Since various laboratory systems have been subjected to various SANS experiments, such as the work by Mildner and Carpenter (1984), the pinhole collimation geometry is an important aspect of SANS measurements.

For a biconcave lens made of MgF\(_2\) with \(R = 2.5\,\text{cm}\), \(\xi = 1.6 \times 10^{-4}\) for 10 Å neutrons and \(f = 156\,\text{m}\). Therefore, for the 30 m SANS instrument at NIST, with \(L_1 = L_2 = 15\,\text{m}\), 21 MgF\(_2\) lenses are needed to focus 10 Å neutrons.

2. SANS Collimation Geometry

Figure 1 a) shows a schematic diagram of the conventional pinhole collimation used in long flight-path SANS instruments. The source and sample apertures, and the distances \(L_1\) and \(L_2\) define the beam size at the detector position. For \(L_1 = L_2\), the optimal pinhole condition is \(A_{\text{p}} = 2A_{\text{p}}\) (Mildner & Carpenter, 1984), and, therefore, the beam profile at the detector is approximately triangular with a base width, \(B_0 = 2A_{\text{p}}\). In this case, the minimum accessible scattering vector, \(Q\), is

\[
Q_{\text{min}} = k\theta_{\text{min}} = c_{\text{p}}A_{\text{p}}/L_2
\]

where \(k = 2\pi/\lambda\), \(A_{\text{p}}\) is the diameter of source aperture and \(c_{\text{p}} = 1\) in ideal case is a smearing factor which includes detector resolution, beam spreading due to gravity, and parasitic scattering. The beam intensity (n/sec) on sample \(I_p\) is given as

\[
I_p(n/\text{sec}) = \frac{d^2\phi}{d\Omega d\lambda} \delta(\lambda) \left( \frac{A_{\text{source}}A_{\text{sample}}}{L^2_1} \right)
\]

where \(\phi\) is the neutron flux, \(A_{\text{source}} = A_{\text{p}}/4\) and \(A_{\text{sample}} = A_{\text{p}}/4 = A_{\text{p}}/16\). Using equation (2) and \(L_1 = L_2\), equation (3) can be rewritten as

\[
I_p = \frac{d^2\phi}{d\Omega d\lambda} \delta(\lambda) \left( \frac{2L}{8} \right)^2 \left( \frac{Q_{\text{min}}}{c_{\text{p}}k} \right)^4
\]

Figure 1 b) shows a schematic diagram of the focusing lens geometry. When \(L_1 = L_2\), the size of a perfectly focused beam at the detector is same as the source size, \(B_0 = A_{\text{p}}\). It should be noticed that the beam size at the detector is independent of the sample aperture size. Therefore, sample sizes up to the size of the lens opening can be...
In the ideal case where the intensity on sample is proportional to \(Q^4\), the gain provided by the lenses compared with pinhole collimation can be defined as the ratio of the intensities on sample. From equations (4) and (5), the gain can be expressed as

\[
\text{Gain} = \frac{I_L}{I_p} = \left(\frac{A_{\text{L}}}{A_{\text{LP}}}\right)^2 \left(\frac{A_{\text{L}}}{A_{\text{LP}}}\right)^2 \left(\frac{c_p}{c_L}\right)^2 T_L .
\]

Figure 1
Beam collimation geometry for SANS. a) Pinhole collimation geometry, b) Focusing lens geometry.

used to increase intensity without affecting the beam size at the detector. In this lens configuration, the minimum \(Q\) is given as

\[
Q_{\text{min}} = c_L k \frac{A_{\text{L}}/2}{L_2} \tag{5}
\]

where \(c_L\) is a smearing factor which includes detector resolution, chromatic aberration, gravity effect, and any scattering by the lenses. It can easily be shown that the intensity on sample can be expressed as

\[
I_L = T_L^2 \frac{d^2\phi}{d\Omega d\lambda} \delta\lambda \left(\frac{\pi A_{\text{L}}}{c_L} \right)^2 \left(\frac{Q_{\text{min}}}{c_L k}\right)^2 \tag{6}
\]

where \(T_L\) is the neutron transmission of the lenses.

From equations (4) and (6), it should be noticed that \(I_L\) is proportional to \(Q_{\text{min}}^2\) while \(I_p\) is proportional to \(Q_{\text{min}}^4\). Therefore, as \(Q_{\text{min}}\) becomes smaller, the intensity on sample decreases in both cases, but much faster for pinhole collimation than for focusing lens collimation. Therefore, there is a crossover at a certain value of \(Q_{\text{min}}\) below which focusing outperforms pinhole collimation. For a given \(Q_{\text{min}}\), the gain provided by the lenses compared with pinhole collimation can be defined as the ratio of the intensities on the sample. From equations (4) and (5),

\[
\text{Gain} = \frac{I_L}{I_p} = \left(\frac{A_{\text{L}}}{A_{\text{LP}}}\right)^2 \left(\frac{A_{\text{L}}}{A_{\text{LP}}}\right)^2 \left(\frac{c_p}{c_L}\right)^2 T_L .
\]

In the ideal case where \(c_p = c_L = 1\) and \(T_L = 1\), the gain is simply a product of the squares of the ratios of aperture diameters. For example, when \(L_2 = L_4\), to make a 2 cm diameter beam at the detector, in the lens configuration we can use \(A_{\text{L}} = 2\) cm and \(A_{\text{LP}} = 1.5\) cm while in the pinhole collimation we need to use \(A_{\text{LP}} = 1\) cm and \(A_{\text{LP}} = 0.5\) cm. Then the gain is \(4 \times 9 = 36\). In practice, this ideal gain will be reduced by the intensity attenuation by the lenses and the smearing factors.

The focusing characteristics of a set of 28 biconcave MgF\(_2\) lenses, including the effects of chromatic aberration due to the wavelength spread in the incident beam, and distortion due to gravity, have been extensively studied by MC simulations and measurements on one of the 30 m SANS instruments at NIST. Here we present a case where \(\lambda = 8.15\) Å, \(\Delta \lambda/\lambda = 11\%\), \(L_1 = 15.5\) m, \(L_2 = 15.8\) m, \(A_1 = 0.95\) cm and \(A_2 = 1.59\) cm were used. The MC simulations of the beam intensity distributions at the detector position are presented in Figure 2 a) (without lenses) and Figure 2 b) (with 28 MgF\(_2\) biconcave lenses that have a radius of curvature of 2.5 cm and a center thickness of 1 mm). In this simulation, a triangular wavelength distribution is assumed and the gravity effect is included. It can be clearly seen that the beam with the lenses is much smaller than one without the lenses and is more intense (8.5 times) at the center of the beam. The size of the focused beam should be same as the source aperture size, but the beam is somewhat larger than the source aperture for two reasons. First, chromatic aberration due to the wavelength spread of the incident beam appears in both the horizontal and vertical directions. For neutrons with shorter (or longer) wavelengths than the mean wavelength, which satisfies the Gaussian optics relation, the detector position at \(L_1\) is before (or after) the focal plane and the beam at \(L_2\) is larger than the source aperture. Second, the beam is elongated along the vertical direction due to gravity. The short wavelength neutrons appear at the top and the longer wavelength neutrons at the bottom.

In order to measure the detailed shape of the focused beam intensity, a high resolution (± 0.05 mm), wide dynamic range (greater than 5 orders of magnitude), imaging technique was utilized: an image transfer technique based on neutron activation of dysprosium foil followed by off-line exposure of a gamma-ray-sensitive image plate. Therefore the beam intensity at the focal plane could be measured over 5 orders of magnitude, essentially free of any smearing due to finite detector resolution. Figure 2 c) shows a 2D image of the focused beam measured with the image transfer technique. In this measurement, all conditions were the same as those used in the MC simulation presented in Figure 2 b). The measured 2D distribution of the focused beam is qualitatively very close to the MC simulation.

To understand the properties of the focused beam in a more quantitative way, 1D profiles across the peak intensity region were obtained from the 2D images. Figure 3 shows the 1D profiles along the vertical direction. A high signal-to-noise ratio (ratio of the peak intensity to the base line intensity) is important for the focused beam to be useful for SANS experiments. To determine the signal-to-noise ratio and the detailed shape at the tail, the focused beam profiles were examined on log scale. The measured profile shows a signal-to-noise ratio greater than \(10^7\) which is very suitable for SANS experiments. The agreement between the measured and the simulated profiles is quite good down to the intensity level \(10^7\), below which the measured profile shows a wider beam width than the simulated one. This deviation can be explained by three factors. First, the bump at \(Y = 0\) mm position is due to fast neutrons coming from the reactor. Second, while in the simulation a perfect triangular wavelength distribution was assumed, the actual wavelength distribution has an additional tail beyond the cut off of a triangular distribution (Glinka et al., 1998) which contributes a further chromatic aberration and spreading due to
Figure 2
2D Images of focused and unfocused beams. $\lambda = 8.15\text{Å}, \Delta \lambda / \lambda = 11\%$, $L_1 = 15.5$ m, $L_2 = 15.8$ m, $A_1 = 0.95$ cm and $A_2 = 1.59$ cm. a) Unfocused beam: MC simulation. b) Focused beam with 28 MgF$_2$ biconcave lenses: MC simulation. c) Focused beam with 28 MgF$_2$ biconcave lenses: measured with image transfer technique. The size of boxes is 40 mm x 40 mm.

gavity. Third, the lenses produce small-angle scattering but, from a practical point of view, it is negligible. The width of the simulated beam at the $10^{-4}$ intensity level is 1.89 cm which is twice the size of the source aperture (0.95 cm). Of the total enlargement of 0.94 cm, 0.66 cm (determined from a horizontal profile that has no broadening due to gravity) is due to the chromatic aberration and 0.28 cm is due to gravity.

Figure 3
1D profiles of focused beam images in figure 2 along the vertical direction. $\lambda = 8.15\text{Å}, \Delta \lambda / \lambda = 11\%$, $L_1 = 15.5$ m, $L_2 = 15.8$ m, $A_1 = 0.95$ cm and $A_2 = 1.59$ cm. 28 MgF$_2$ biconcave lenses were used.

3. Intensity Gain of the Focusing Lens Collimation
We have measured the beam intensity profiles for the lens and pinhole configurations under conditions that produce the same ideal $Q_{\min}$. From these we have determined the intensity gain as defined in equation (7). Since, for a given wavelength and the detector distance, $Q_{\min}$ is proportional to the beam size at the detector, measurement conditions were chosen to give the same ideal beam size at the detector: pinhole collimation with $A_\rho = 0.95$ cm and $A_\varphi = 0.48$ cm, and a focusing lens configuration with 28 MgF$_2$ lenses, $A_\rho = 1.91$ cm, and $A_\varphi = 1.43$ cm. In both cases the ideal beam size at the detector is 1.91 cm. Here it should be noticed that $A_\rho = 2 A_\varphi$ and $A_\varphi = 3 A_\rho$. All other conditions were kept same in both cases, $\lambda = 8.44\text{Å}, \Delta \lambda / \lambda = 11\%$ and $L_1 = L_2 = 15.7$ m.

In this case, the beam intensity profiles were measured with the SANS instrument's He$^3$ 2D detector (spatial resolution ~ 0.5 cm). The measurements were made in two steps due to the limited dynamic range of this detector. First, the central regions of the profiles were measured with a plexiglass beam attenuator (with a known attenuation factor) placed right before the source aperture. Second, the wings of the profiles were measured with the attenuator removed and a 2.54 cm beamstop placed at the center of the detector. The central (rescaled with the attenuation factor) and wing profiles were joined to give the full intensity profiles.

Figure 4 shows the intensity profiles for the lens and the pinhole measurements. In both cases, the signal to noise ratio is about $10^4$ at the points indicated by the arrows in the figure. The total intensity gain of the lens configuration compared with the pinhole collimation is 26, which is nearly equal to the product of the squares of the ratios of aperture diameters multiplied by the neutron transmission of the lenses ($T_L = 0.72$). To make a fair comparison, however, we need to consider the actual beam sizes. The lens collimation results in a slightly larger beam than the pinhole collimation, which can be
quantified in terms of the smearing factors, \( c_1 = 1.91 \) and \( c_2 = 2.45 \) at the signal-to-noise ratio of \( 10^{-7} \). Using equations (2) and (5), \( Q_{\text{min}} = 0.00086 \ \text{Å}^{-1} \) for the pinhole configuration and \( Q_{\text{min}} = 0.0011 \ \text{Å}^{-1} \) for the lens configuration. If we consider the smearing factors and \( Q_{\text{min}} \), the actual gain of the lens configuration is less than 26.

Figure 5
Estimated intensity gain as a function of \( Q_{\text{min}} \). The measured smearing factors, \( c_1 = 1.91 \) and \( c_2 = 2.45 \) for SNR = \( 10^{-7} \), were used. \( \lambda = 8.44 \ \text{Å}, \Delta \lambda/\lambda = 0.11\% \), \( L_1 = L_2 = 15.7 \ \text{m}, A_\perp = 2.1 \ \text{A}^{-1}, \text{and} A_\parallel = 1.59 \ \text{cm} \). The crossover point is \( Q_{\text{min}} = 0.0038 \ \text{Å}^{-1} \).

Figure 6
Comparison of the scattering intensities from an aluminium sample. For both the lens and pinhole configurations, \( \lambda = 8.44 \ \text{Å}, \Delta \lambda/\lambda = 0.11\% \), and \( L_1 = L_2 = 15.7 \ \text{m}, A_\perp = 0.95 \ \text{cm}, A_\parallel = 0.48 \ \text{cm}, A_\perp = 1.91 \ \text{cm}, \text{and} A_\parallel = 1.43 \ \text{cm} \). 28 MgF\(_2\) lenses were used. The scattering intensity for the lens configuration is about 25 times higher than for the pinhole configuration. The first valid data point is \( Q = 0.0011 \ \text{Å}^{-1} \).

4. Conclusion
The focusing properties of multiple biconcave MgF\(_2\) lenses have been shown to extend the lower limit of \( Q \) in long-flight-path small angle neutron scattering instruments. The signal-to-noise ratio of cold neutrons focused by 28 MgF\(_2\) lenses is greater than \( 10^7 \), the small angle scattering from the lenses is negligible and the beam attenuation by the lenses is manageable. The focusing lens configuration outperforms conventional pinhole collimation at, for the NIST 30m SANS instruments, \( Q_{\text{min}} \) below \( 0.004 \ \text{Å}^{-1} \). Compared with pinhole collimation, the intensity gain of the lens configuration at \( Q_{\text{min}} \) around \( 0.001 \ \text{Å}^{-1} \) is greater than 1 order of magnitude. This gain extends the lower limit of the accessible \( Q \) and would make many previously impractical SANS measurements possible.

This work is based upon activities supported by the National Science Foundation under Agreement No. DMR-9423101. The names of commercial items in this report are included solely to provide a complete description of the experimental methods and do not constitute or imply endorsement by the National Institute of Standards and technology for any purpose.

References