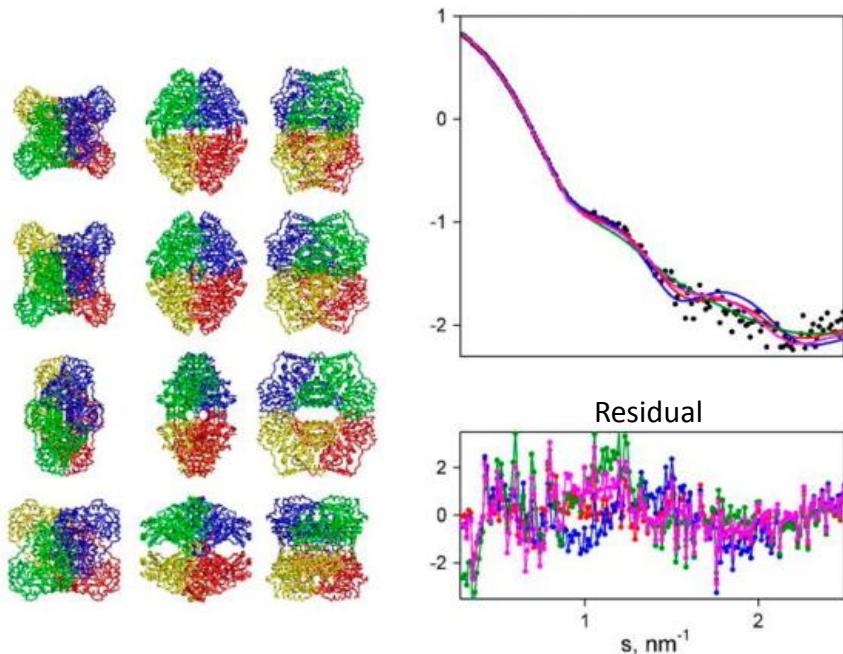


SANS Inversion Via Magnetic Tagging

Kathryn Krycka, Erik Brok, Chuck Majkrzak

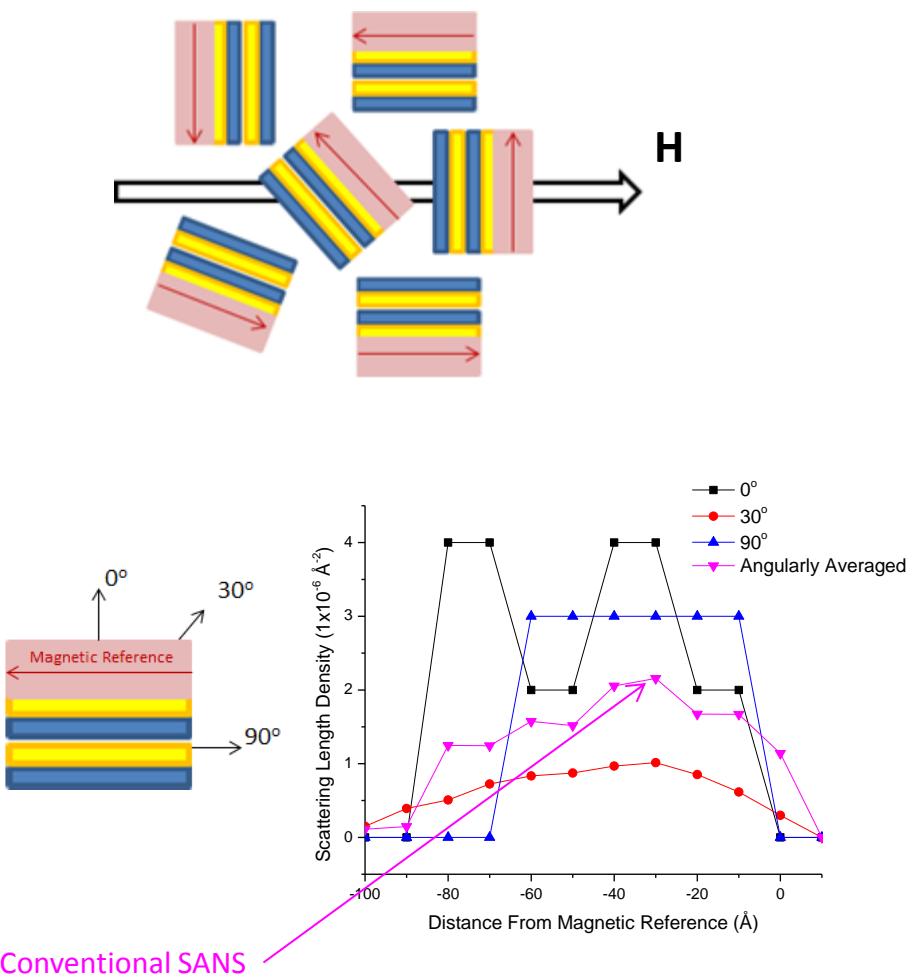
Fixed-Tag, Phase Recovery: “Killing two birds with one stone”

(A) Obtain unique scattering amplitude (magnitude + phase), rather than potentially non-unique scattering intensity (magnitude squared)

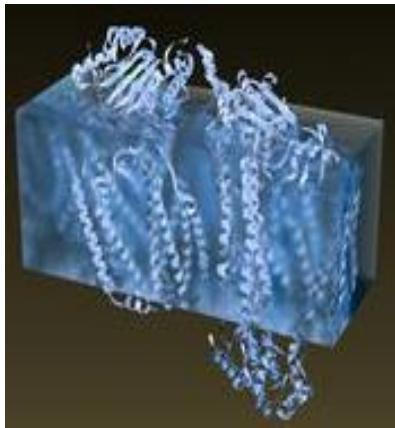


M.V. Petoukhov and D.I. Svergun, Biophysical Journal, **89**, 1237–1250 (2005).

(B) Reference fixed relative to sample
additionally removed the SANS Rotational
Averaging (shown in 2D)

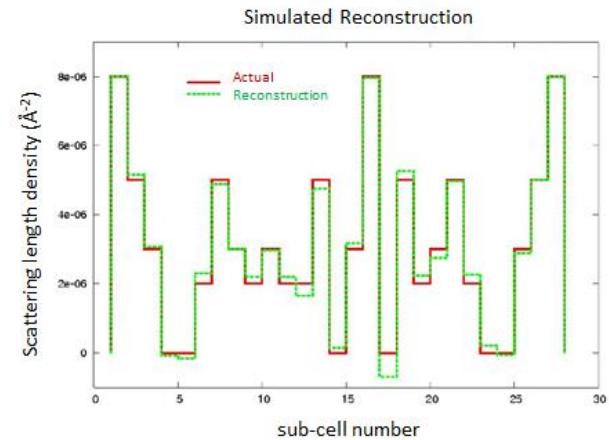
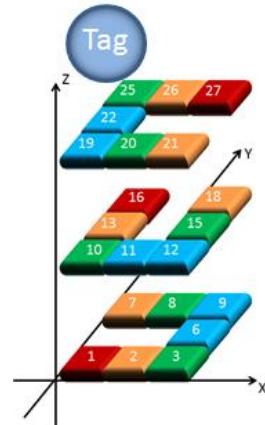
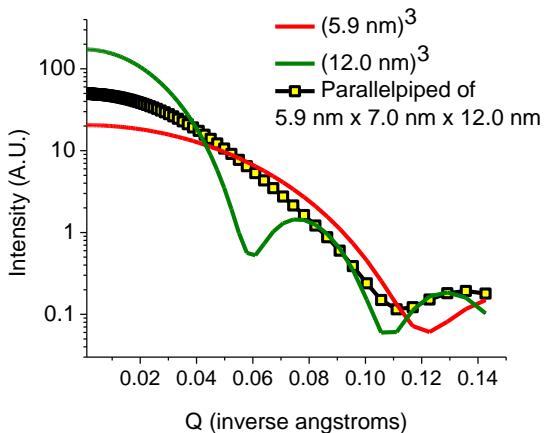


Eventual Goal (3D Inversion)



K. A. Robinson, C. Pokalsky,
S. Kruegar, L. J. Prochaska,
Protein J., 32, 27-38, 2013

- Quantifying complex 3D structures with non-uniform dimensions is challenging in conventional SANS
- C. oxidase in which parameters of 5.9nm x 7.0nm x 12.0nm were carefully determined by comparing many possible models



Phase-sensitive small angle neutron scattering, C.F. Majkrzak, K. Krycka, S. Krueger, N.F. Berk, P. Kienzle and B. Maranville, J. Appl. Cryst. 47, 780-787, 2014

- Tag(s) should be asymmetric in X,Y,Z dimensions for maximum phase retrieval
- Tags must attach at same point on objects of interest (one-to-one)
- Tag-Object bond should be “relatively stiff”
- “Sufficient Q range with sufficiently low noise”

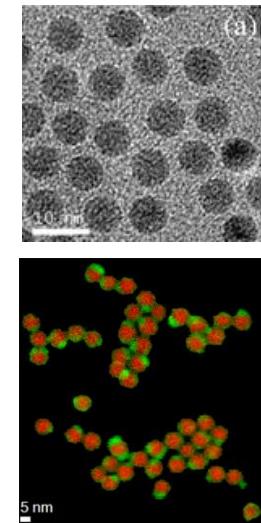
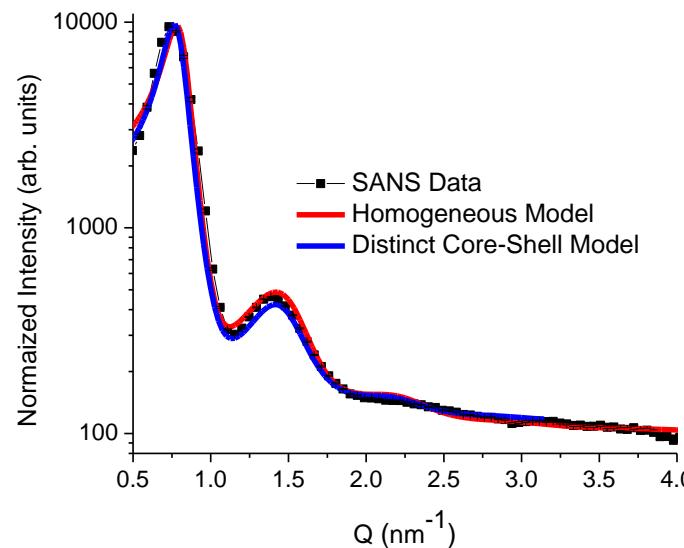
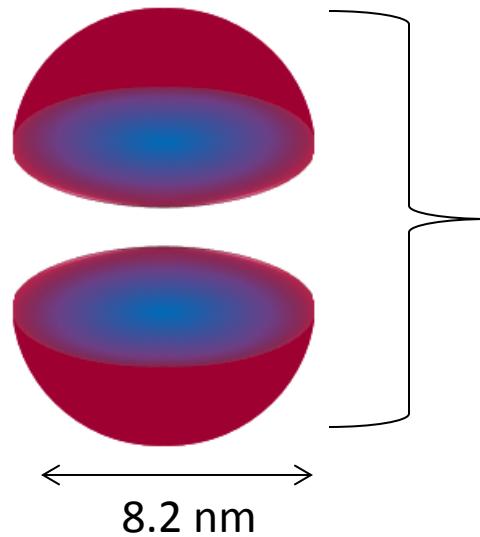
Starting with 1D: Even Core-Shell Morphology Is Difficult To Resolve

Example:

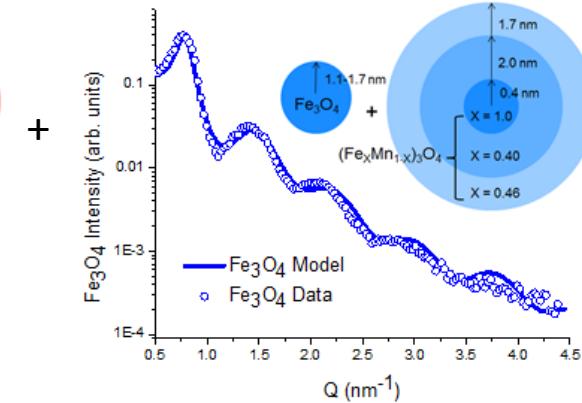
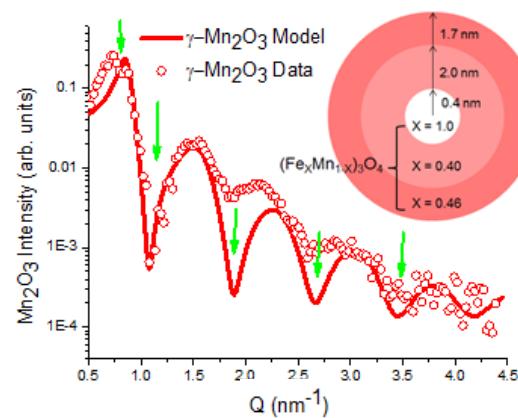
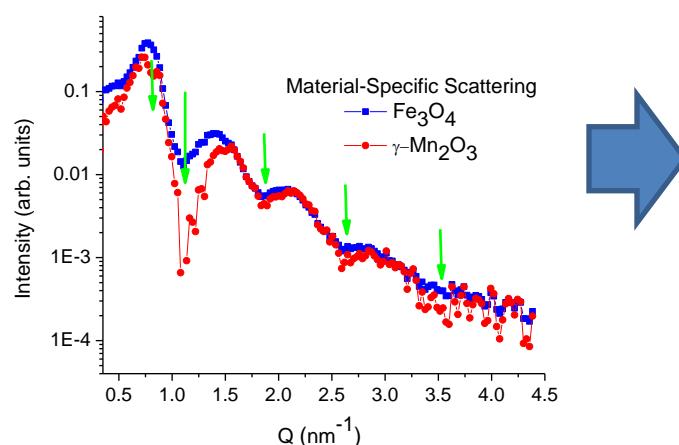
Fe_3O_4
SLD = $6.97 \times 10^{-6} \text{ Å}^{-1}$

$\gamma\text{-Mn}_2\text{O}_3$
SLD = $1.71 \times 10^{-6} \text{ Å}^{-1}$

More than a factor
of 4 different!



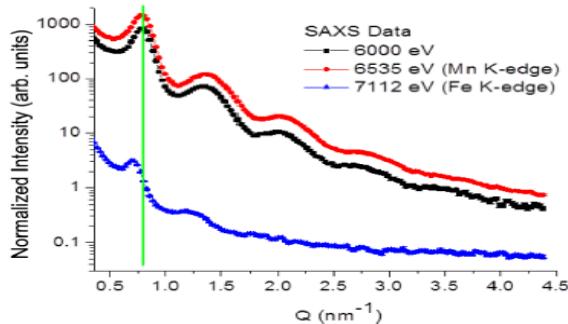
Obtain *phase information* from
resonant x-ray scattering:



Details in ACS Nano 7, 921-931 (2013)

Ways to Recover Phase

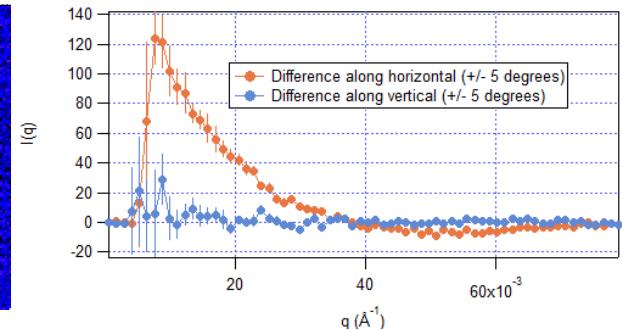
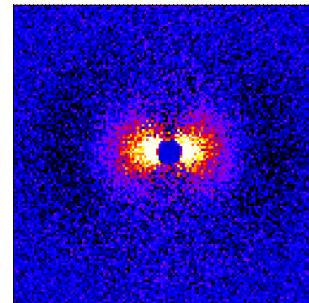
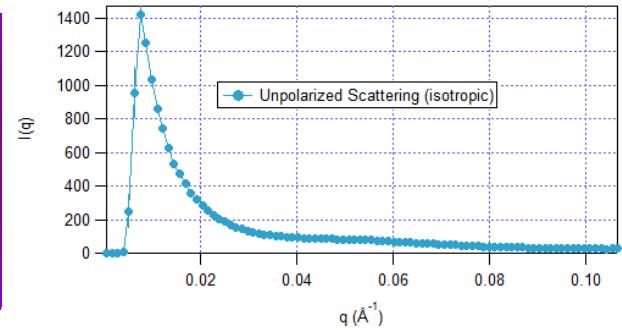
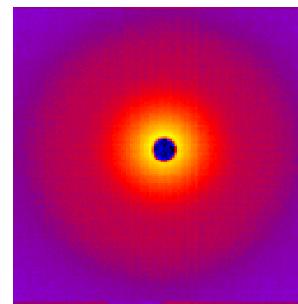
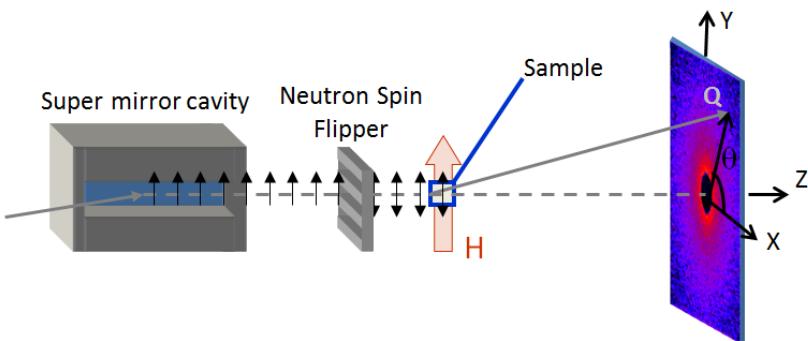
1. Resonant X-ray Scattering (typically won't remove angular averaging)



2. H-D substitution of solvent (neutrons, angular averaging still an issue)

3. Isomeric substitution (x-ray or neutron, question about changing sample, typically applied to powder or single crystals)

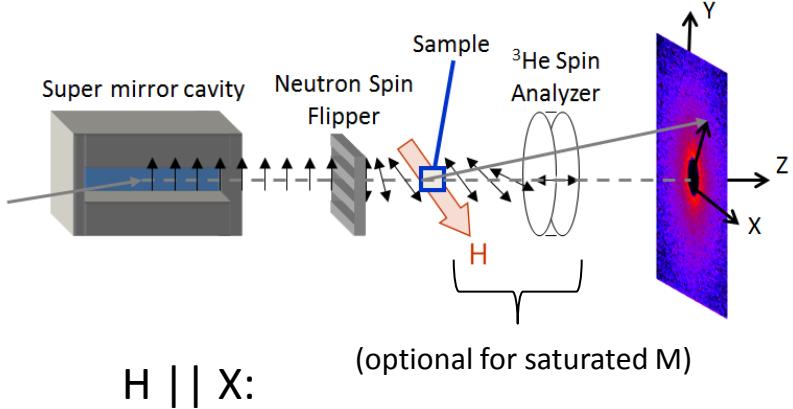
4. Single-sample magnetic reference tag with polarized neutron beam



- ↑, ↓ difference (Nucl-Mag C.T.) eliminates background!
- Neither polarization of optics nor sample saturation need to be perfect in order for this to work.

Recovering Phase with Magnetic Tag

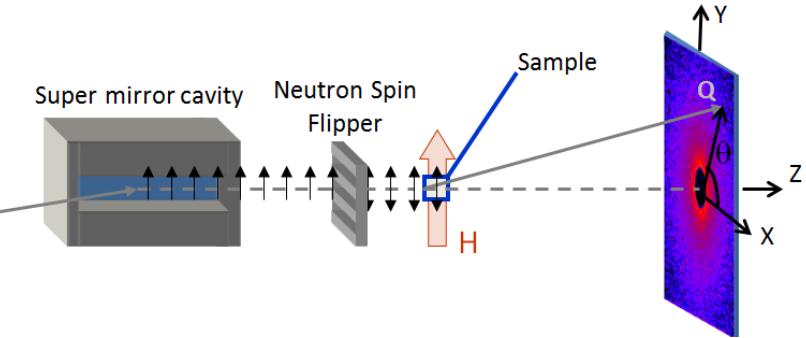
Full-Polarization w/ any magnetic tags



$$\begin{aligned} \sigma_{\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^{\uparrow\downarrow}(\mathbf{Q}) &= N(\mathbf{Q})N^*(\mathbf{Q}) + M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \sin^4(\theta) \\ &+ M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \cos^2(\theta) \sin^2(\theta) \\ &- [M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \\ &+ M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \sin^3(\theta) \cos(\theta) \\ &\pm [N(\mathbf{Q})M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) + N^*(\mathbf{Q})M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \sin^2(\theta) \\ &\mp [N(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) + N^*(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \sin(\theta) \cos(\theta) \end{aligned}$$

$$\begin{aligned} \sigma_{\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^{\uparrow\downarrow}(\mathbf{Q}) &= M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \\ &+ M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \cos^4(\theta) \\ &+ M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \sin^2(\theta) \cos^2(\theta) \\ &- [M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \\ &+ M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q})M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \sin(\theta) \cos^3(\theta) \\ &\pm i[M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) \\ &- M_{x,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q})M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \sin(\theta) \cos(\theta) \\ &\mp i[M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q}) - M_{y,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}^*(\mathbf{Q})M_{z,\hat{\mathbf{p}}_x \perp \hat{\mathbf{n}}}(\mathbf{Q})] \cos^2(\theta) \end{aligned}$$

Half-Pol w/ saturated magnetic tags



$$\begin{aligned} \sigma_{\downarrow,\uparrow} &= NN^* + M_{||H}M_{||H}^* \cos^2(\theta) \\ &+ [-NM_{||H}^* + N^*M_{||H}] \cos(\theta) \end{aligned}$$

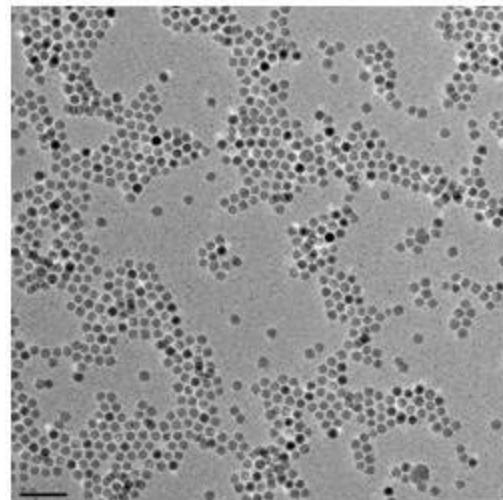
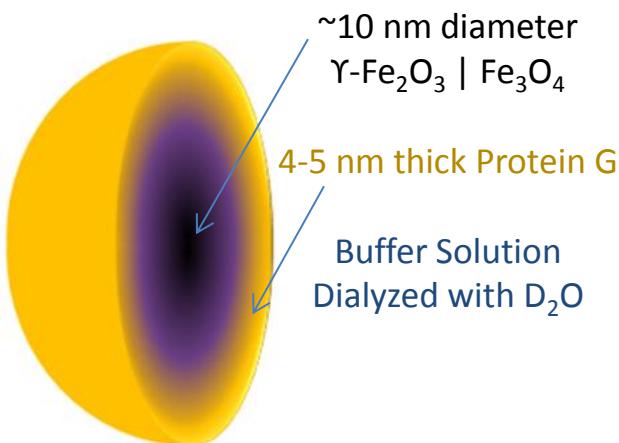
- 1) $NM_{||H}$ is Nuclear-Magnetic Cross Term which provides the phase information for N
- 2) Choose coordinates so M (tag) is *real*
- 3) Only sensitive to objects *coherent* with magnetic tags in the $\downarrow - \uparrow$ difference
- 4) Polarizing mirror ~80% transmission for preferred spin state; polarization > 99%

Project Steps for SANS Tagging + Inversion

- Characterize magnetic tags in solution
 - will incoherent hydrogen scattering be problematic?
 - is dilute limit too weak?
 - does chaining occur at saturating magnetic fields?
- Invert core-shell system (1D) with included magnetic component
 - what minimum Q do we need to reach for a given real-space resolution?
 - what statistical signal:noise is needed?
 - focus on simulation versus experimental data comparisons
- Attempt to invert core-shell + attached protein with magnetic component
- (Future) Anisotropic tagging schemes
 - multiple spherical attachments
 - single / multiple nanorods
 - rectangular, layered block (single attachment / protein)
- (Future) Anisotropic attachment
 - oriented Au pads

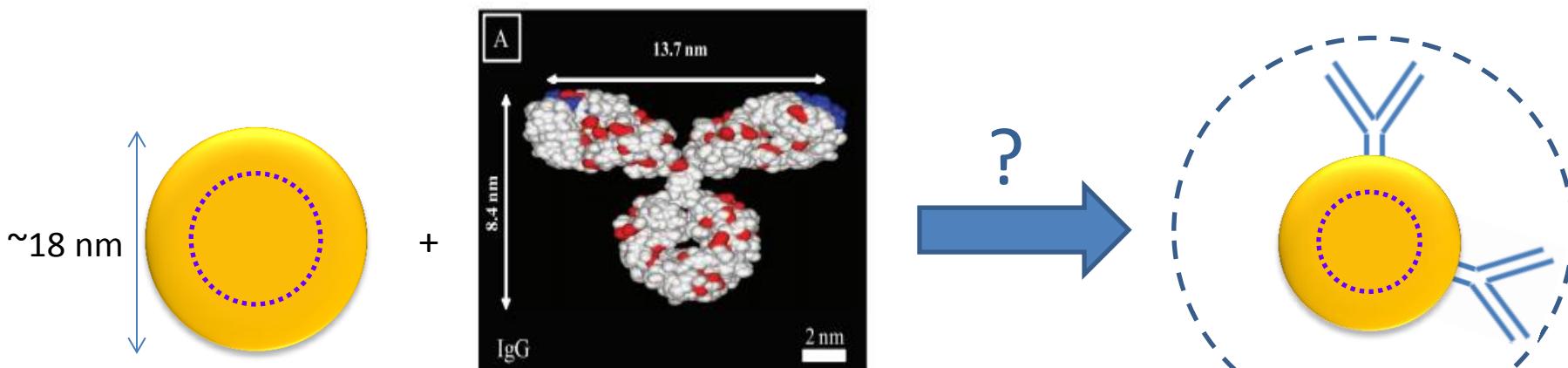
1D Core-Shell Starting Systems

Commercially available from Ocean NanoTech:



TEM of 10 nm Iron Oxide

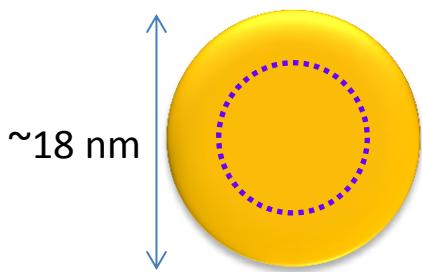
Catalog number:	IPG-10-01
Product name:	Iron oxide nanoparticles with protein G
Solvent:	Borate (0.05M, pH 7.4) with 1 mg/mL BSA, 0.02% NaN ₃
IO size by TEM:	10 nm
Size tolerance:	2.5 nm
Functional group:	Protein G
Storage:	4°C. Do not freeze
pH stability:	4-10
Buffer stability:	Borate, Tris, HEPES, PBS, etc.
Shelf life:	3 months
Volume:	1 mL
Concentration:	1 mg/mL (Fe)
Concentration:	0.86 nmole/mL (nanoparticles)



Preparation and characterization of samples

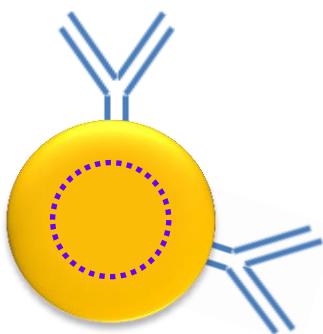
IPG particles:

Stable suspension
Relatively monodisperse
DLS gives small particle size

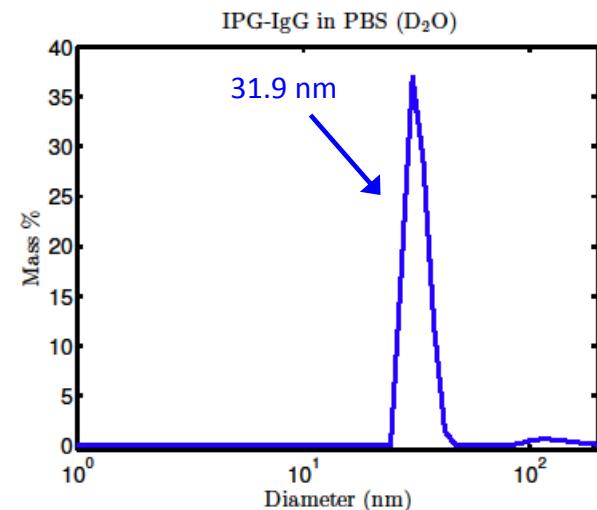
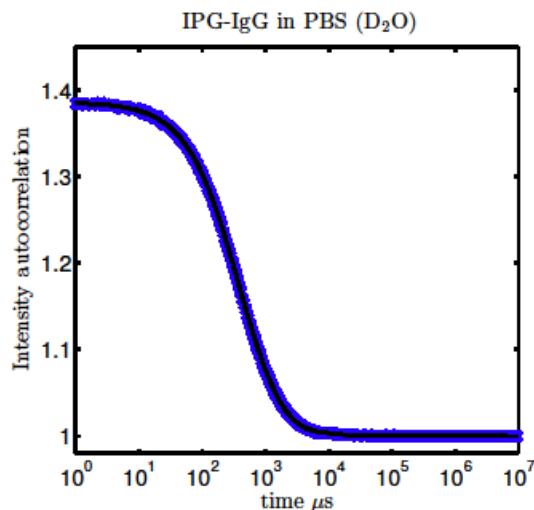
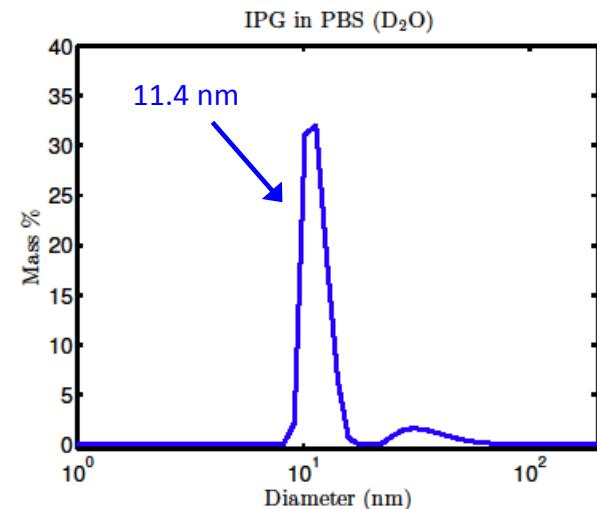
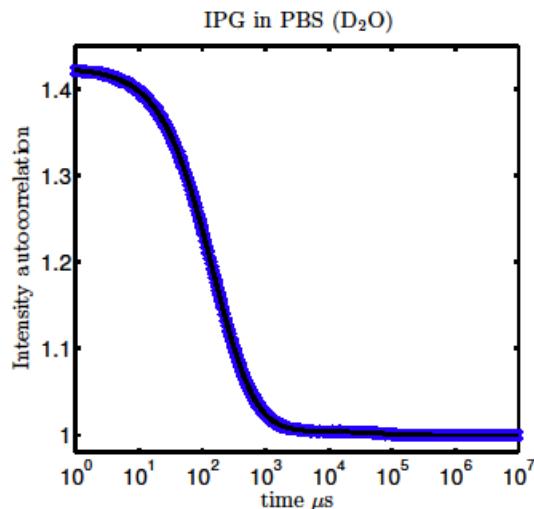


IPG particles – IgG antibodies:

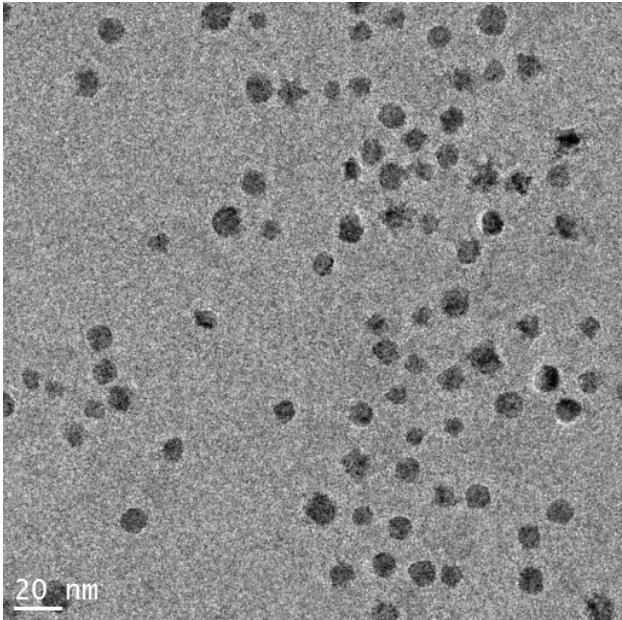
Stable suspension
Relatively monodisperse
No large aggregates



Dynamic light scattering



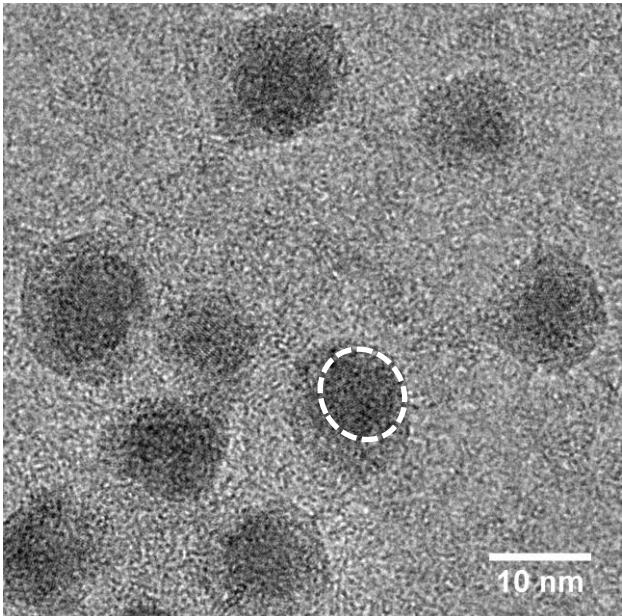
TEM characterization of IPG particles



Well dispersed

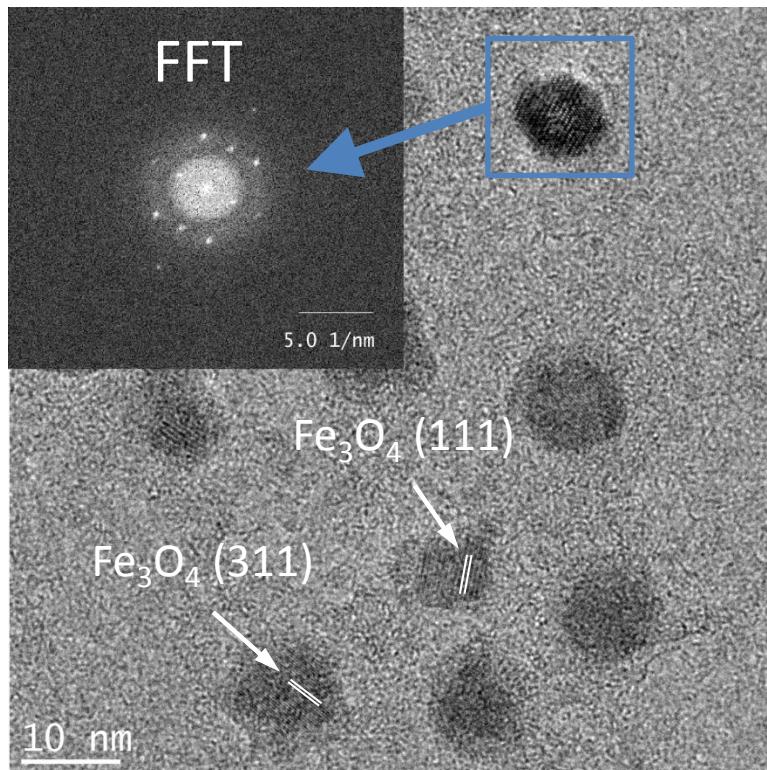
Monodisperse

Spherical



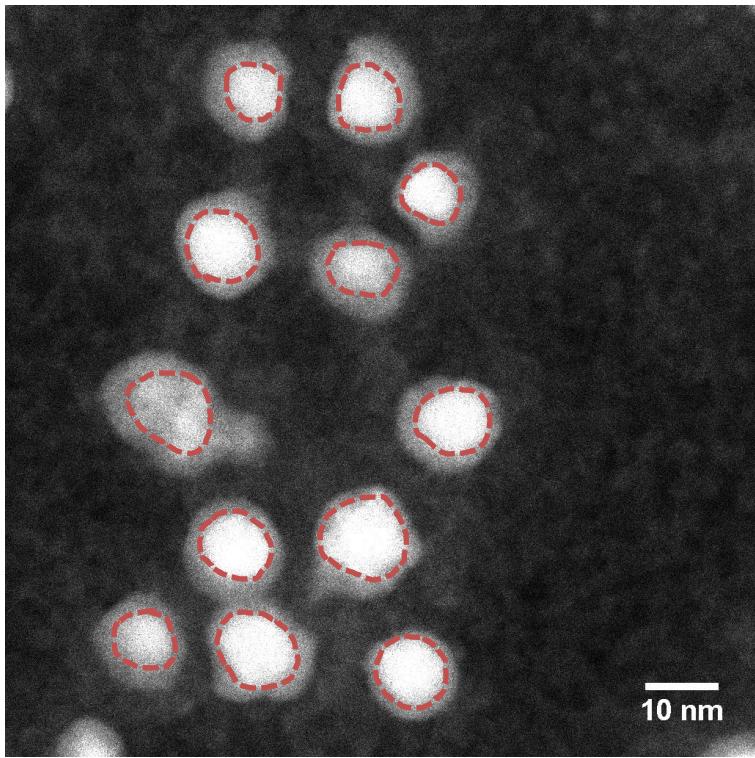
\approx 10 nm core
 \approx 2-5 nm shell

Fe_3O_4 crystal structure



STEM characterization of IPG particles

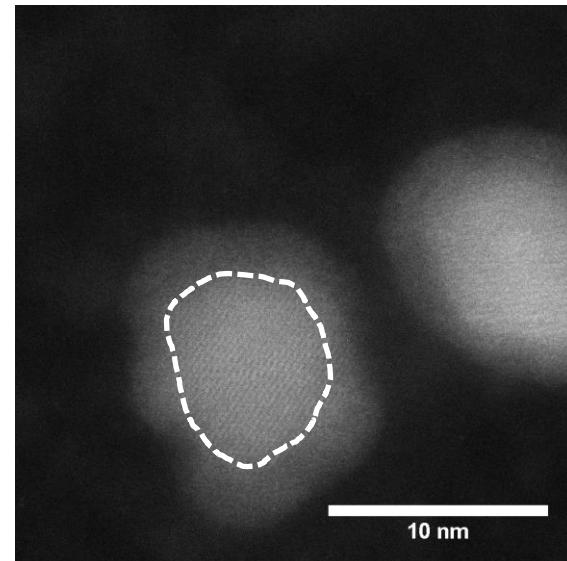
Core/shell contrast more clear in STEM



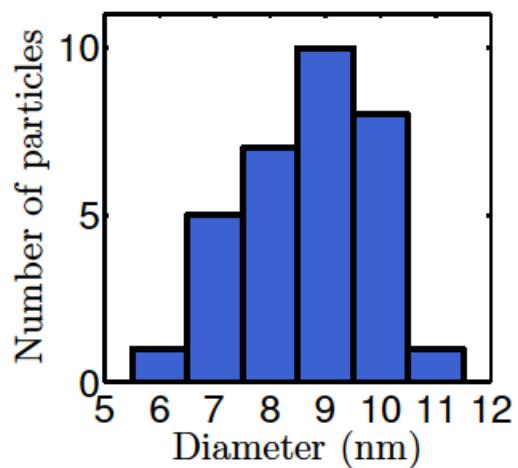
Average core size 8.9 ± 1.3 nm

Other characterization methods

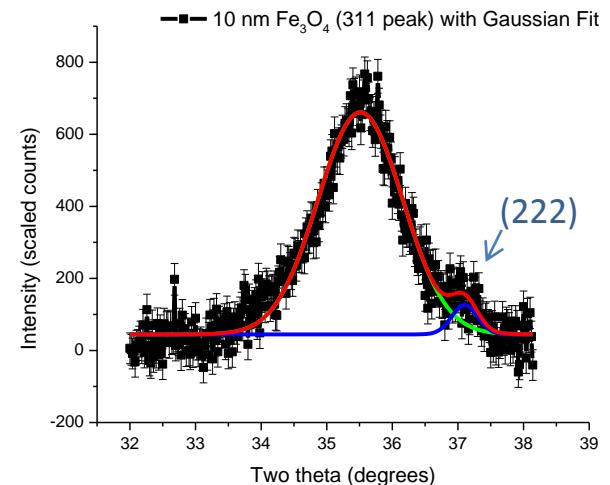
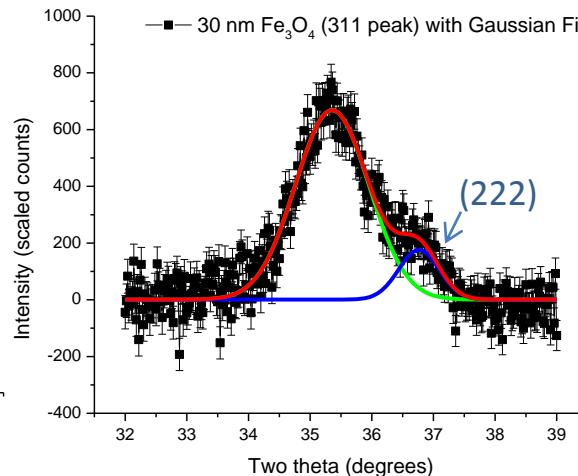
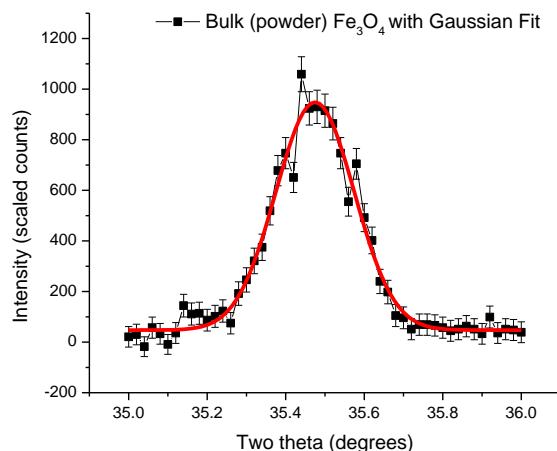
- Dynamic light scattering (hydrodynamic size)
- SAXS (particle core)



Particle core size from STEM



Magnetic Core Characterization : X-Ray Diffraction (sister samples)



$$L = \lambda / \text{FWHM} (\text{radians}) * \cos(2\theta)$$

10 nm (FWHM 1.287°) $\rightarrow 8.4$ nm

30 nm (FWHM 1.166°) $\rightarrow 9.3$ nm

In both cases, crystalline size < magnetic core diameter (see next slide).

Nanoparticles appear to be comprised of multiple crystalline grains.

1D Case: Core-Shell Morphology, Basic Magnetic Characterization

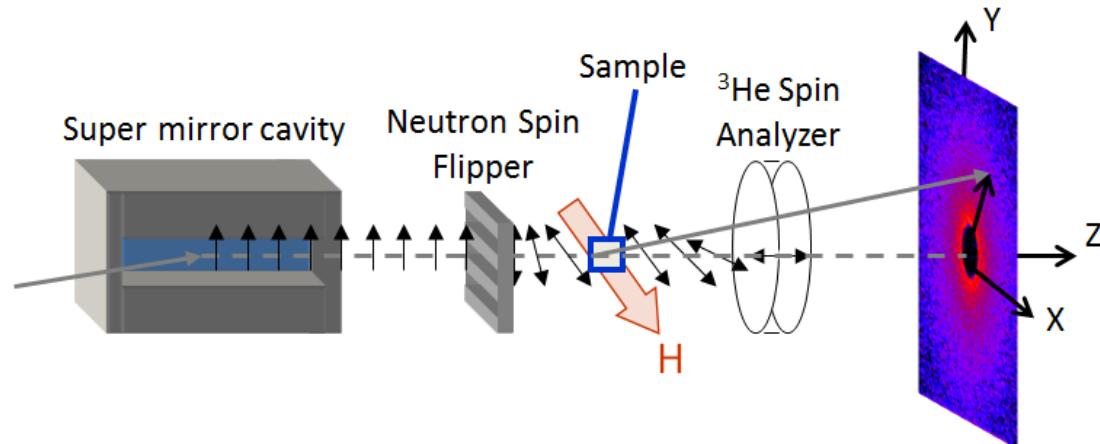
~ 10 nm diameter Fe_3O_4
 $\Delta \text{SLD} = 0.58 \times 10^{-6} \text{ \AA}^{-1}$

4-5 nm thick Protein G
 $\Delta \text{SLD} = -2.0 \times 10^{-6} \text{ \AA}^{-1}$

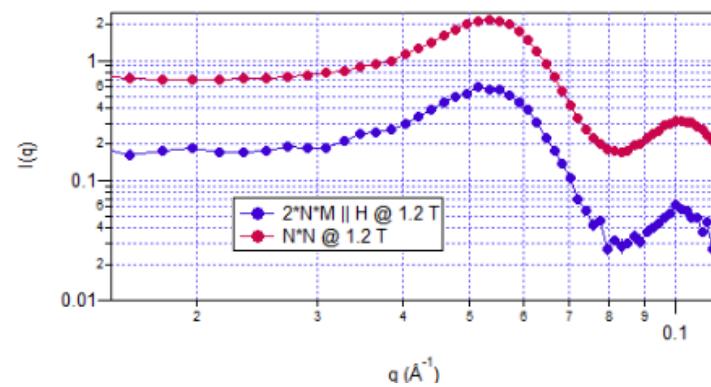
D_2O Buffer
 $\Delta \text{SLD} = 0.0 \times 10^{-6} \text{ \AA}^{-1}$

Dilute Solution
 $\sim 1\text{mg/mL}$

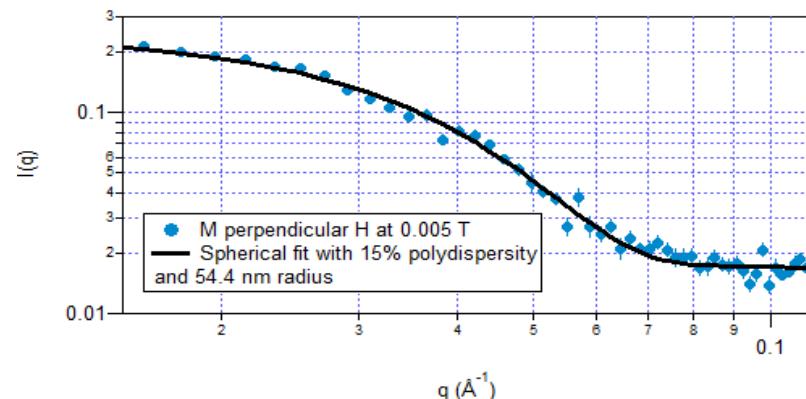
“Full Polarization” at 0.005 T and 1.2 T



‘5% Polydisperse’ within batch Ocean NanoTech Nanoparticles. Examine similar batch (~ 10 nm diameter, dried) Fe_3O_4 cores with thin oleic acid cap to determine diameter and magnetic signal. Thanks to SURF student, Henry Le.

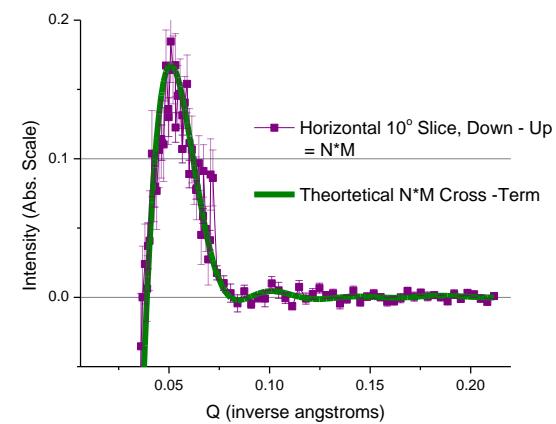
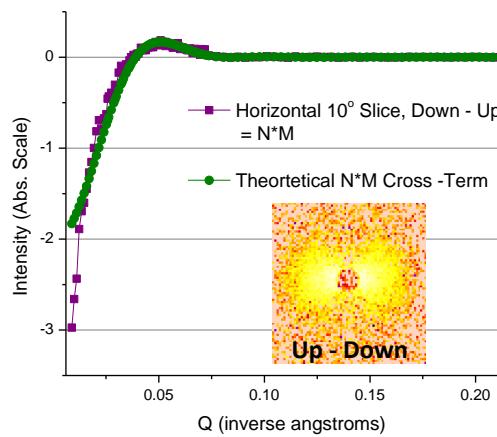
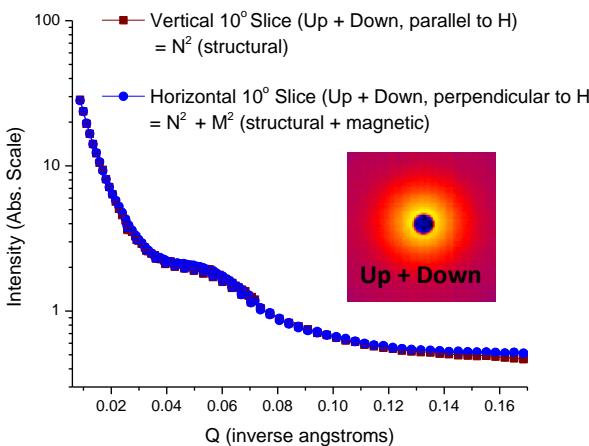
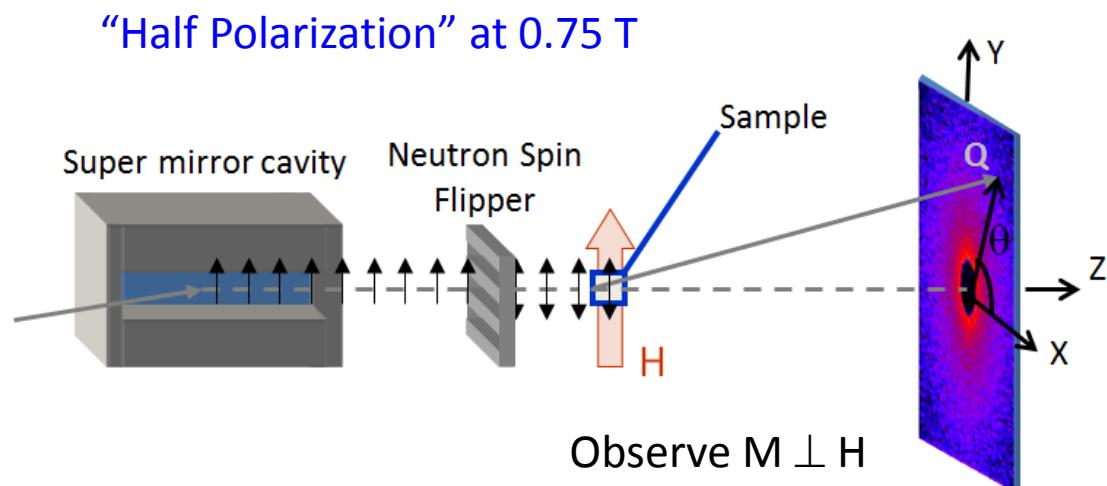
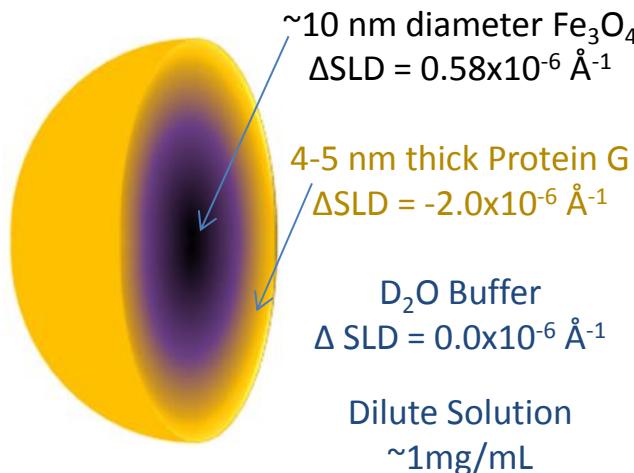


75% as magnetic as bulk Fe_3O_4 at saturation,
300 K (magnetic SLD = $1.1 \times 10^{-6} \text{ \AA}^{-1}$).



Fitting of magnetic signal at remanence gives
magnetic 10.8 nm diameter, PD = 10-15%

1D Case: Core-Shell Morphology With Magnetic Reference

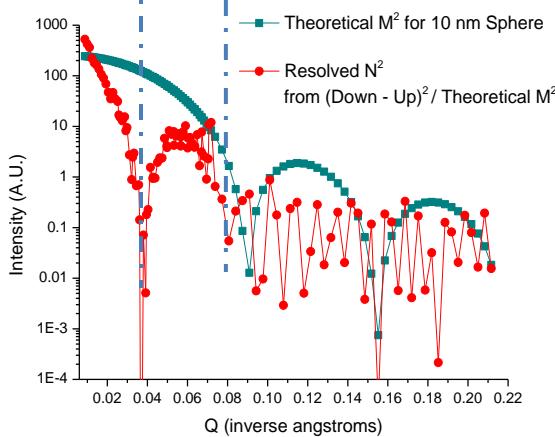
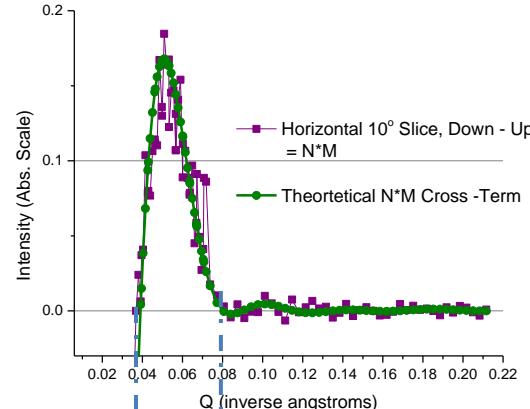


- (1) No apparent chaining due to field
- (2) Hard to detect M^2

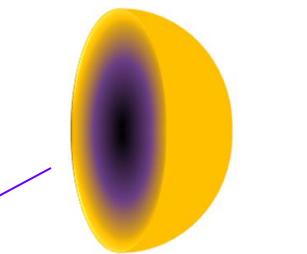
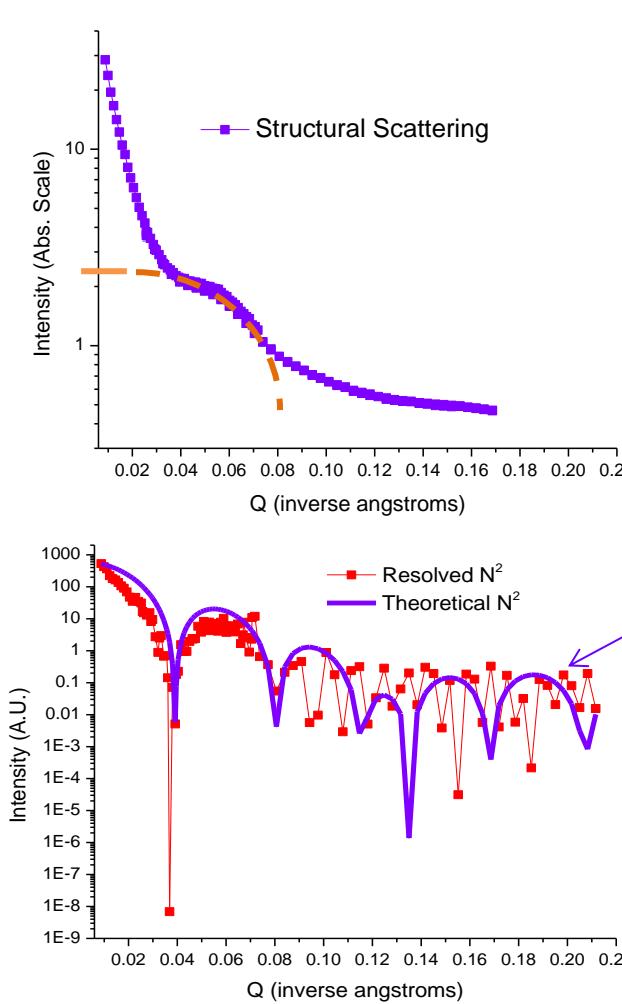
- (1) $N^2 M$ signal is robust (12 hours @ 4m, 12 hours @ 1m)
- (2) Low-Q lenses not possible with pol-beam (USANS, VSANS)
- (3) High-Q currently truncated by magnet shielding
- (4) Note that difference signal completely removes incoherent noise!

1D Case: Quick and Dirty Check (point-by-point analysis)

$$N^2 \propto (\downarrow - \uparrow)^2 / (\text{known } M \text{ scattering}) = (NM)^2 / M^2$$



- Zero crossings in difference correlate to resolved N^2 dips (20 nm total diameter)
- Resolved N^2 is very different from M^2



10 nm diameter Fe_3O_4
 $\Delta \text{SLD} = 0.58 \times 10^{-6} \text{ Å}^{-1}$

5 nm thick Protein G
 $\Delta \text{SLD} = -2.0 \times 10^{-6} \text{ Å}^{-1}$

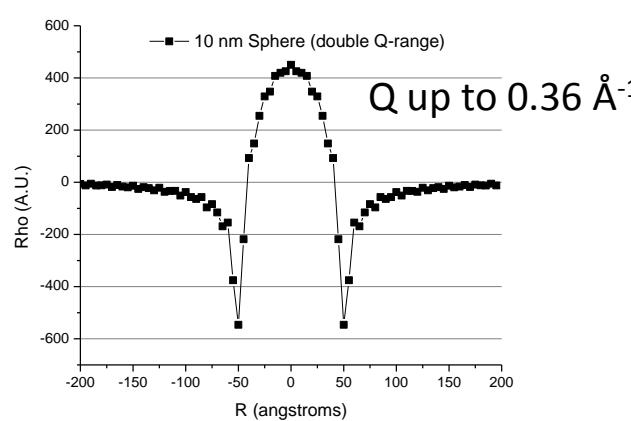
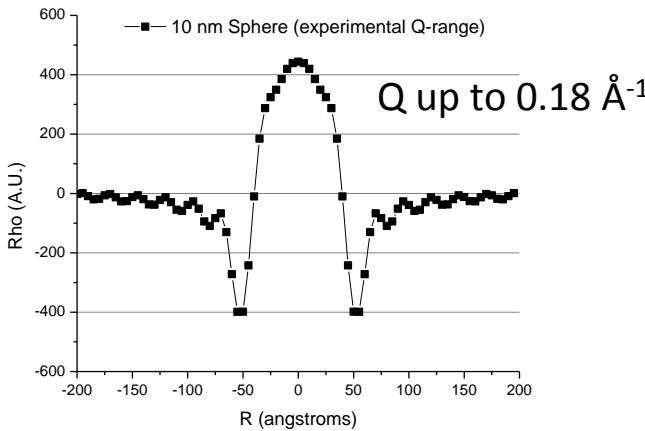
D_2O Buffer
 $\Delta \text{SLD} = 0.0 \times 10^{-6} \text{ Å}^{-1}$

- N^2 details are not easily resolved from unpolarized measurement (top)
- Theoretical and resolved N^2 are very similar

1D Case: Inversion (*Real* nuclear amplitude = Fourier sum of cosines)

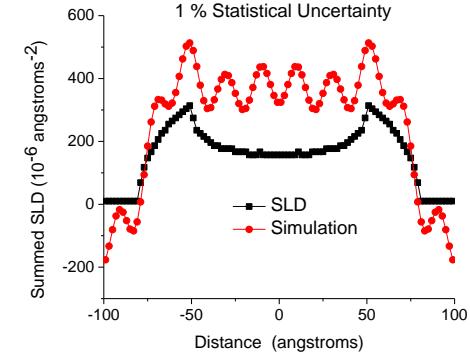
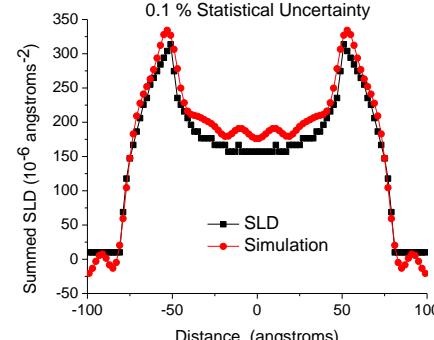
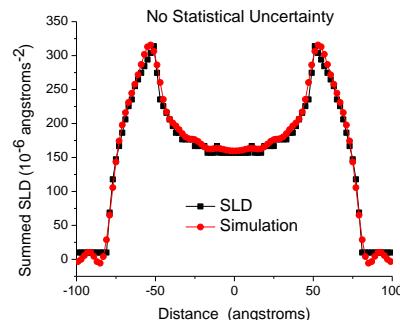
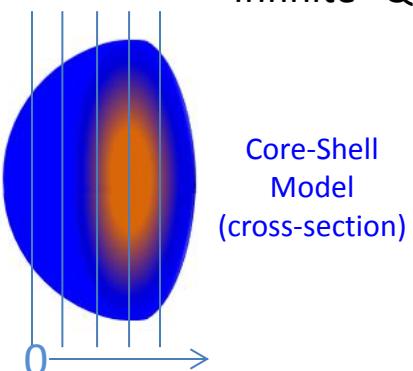
- 1) Since scattering from magnetic reference tag is real, $NM^* + N^*M$ only selects for real component of nuclear scattering amplitude
- 2) $N_{\text{amplitude(real)}} = \sum_n \rho_n [e^{iQ_n R_n}]_{\text{real}} = \sum \rho_n \cos(Q_n R_n)$
- 3) $\rho_n \approx 2/\pi \sum_Q N_{\text{amplitude}} \cos(Q_n R_n) \Delta Q$

Theoretical Inversions From 50 Å Radius Sphere



Ringing at sharp interface edge is expected, especially for finite Q

“Infinite” Q



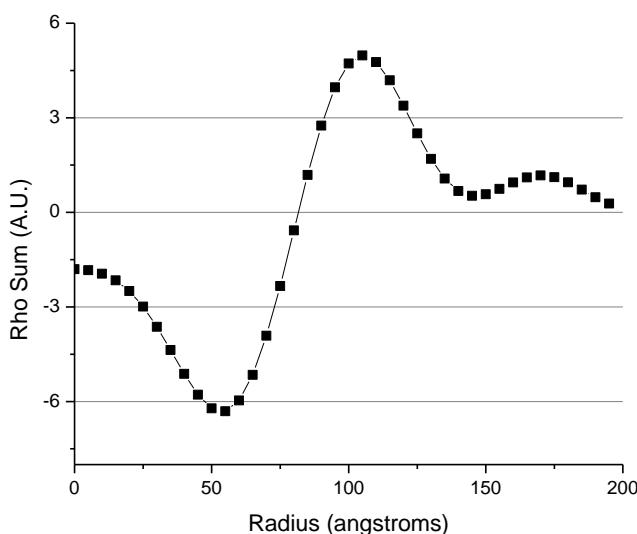
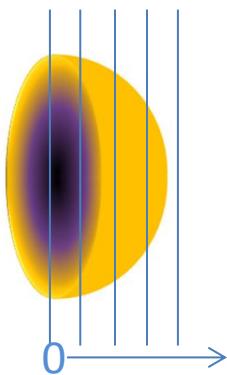
1D Case: Inversion (*Real* nuclear amplitude = Fourier sum of cosines)

Theoretical Inversions (simulation with inversion with Q range similar to experiment)

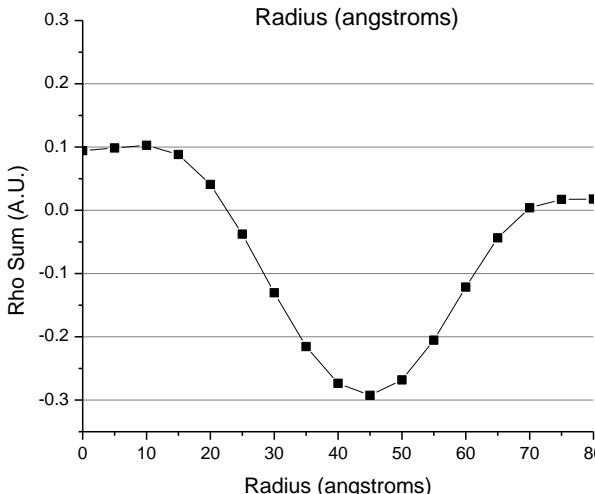
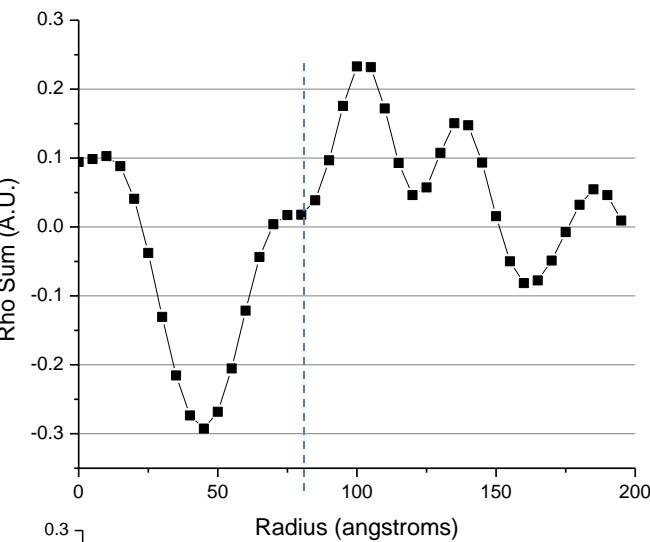
~10 nm diameter Fe_3O_4
 $\Delta \text{SLD} = 0.58 \times 10^{-6} \text{ \AA}^{-1}$

4-5 nm thick Protein G
 $\Delta \text{SLD} = -2.0 \times 10^{-6} \text{ \AA}^{-1}$

D_2O Buffer
 $\Delta \text{SLD} = 0.0 \times 10^{-6} \text{ \AA}^{-1}$

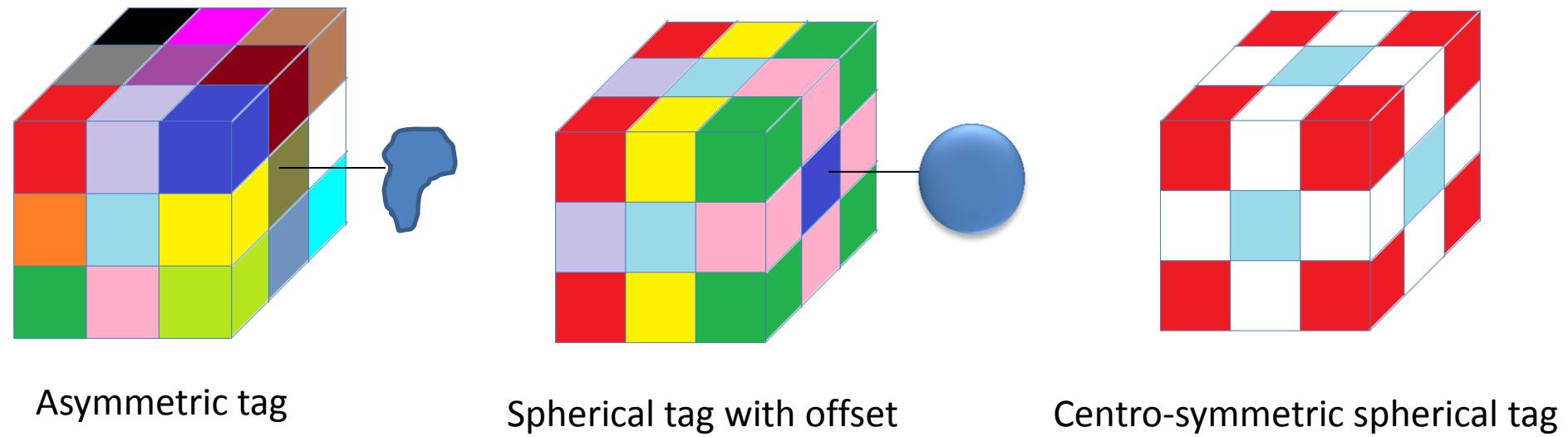


Experimental Inversion from Down – Up Difference
(assuming 50 Å magnetic core radius)



Oscillations are an issue, but profile is plausible and suggestive of a thinner (or less dense) protein shell

1D Case: Inversion in *Spherical Coordinates*

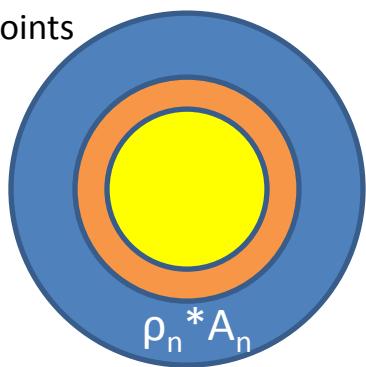


Asymmetric tag

Spherical tag with offset

Centro-symmetric spherical tag

n radial slices
 m q-points

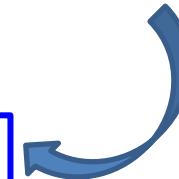


$$A_n(Q) \propto [(\sin(Q*R_n) - Q*R_n * \cos(Q*R_n)) - (\sin(Q*R_{n-1}) - Q*R_{n-1} * \cos(Q*R_{n-1}))] / Q^3$$

$$\begin{pmatrix} A_1(q_1) & \dots & A_n(q_1) \\ \dots & & \dots \\ A_1(q_m) & \dots & A_n(q_m) \end{pmatrix} \begin{pmatrix} \rho_1 \\ \dots \\ \rho_n \end{pmatrix} = (Diff(q_1) \ Diff(q_2) \ \dots \ Diff(q_m))$$

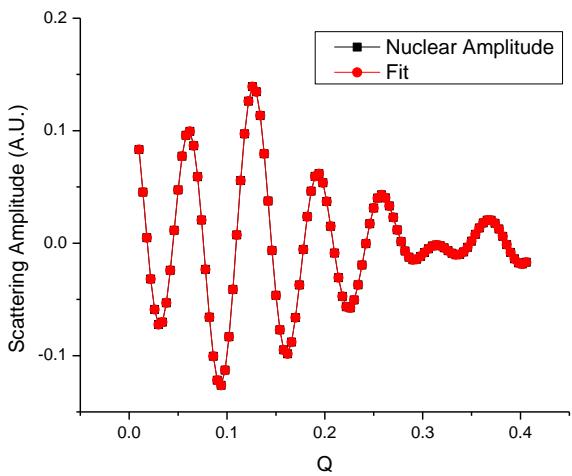
Matrix A (defined)
Matrix ρ (to solve)
Matrix D (measured) =
Nucl-Mag C.T. / 'known' Mag. Amplitude

$$\rho = (A^T A)^{-1} A^T D$$



1D Case: What would it take to achieve 1 nm resolution?

Radius (nm)	Actual_Rho	Rho from Q to 0.4 Å ⁻¹	Rho from Q to 0.3 Å ⁻¹	Rho from Q to 0.25 Å ⁻¹	Rho from Q to 0.2 Å ⁻¹
1	0.5	0.499795	0.501387	4.27271	236.184
2	1.0	1.00005	0.999361	-0.598791	-74.8248
3	2.0	1.99999	2.00043	2.92414	21.3793
4	-2.0	-1.99995	-2.00036	-2.5892	5.38521
5	-3.0	-3.00011	-2.99967	-2.61324	-21.3067
6	5.0	5.0001	4.99976	4.74727	24.6038
7	0.1	0.099975	0.100162	0.260844	-15.7318
8	4.0	3.99996	3.99985	3.9021	14.4677
9	-9.0	-8.99992	-8.99982	-8.94425	-14.7146
10	-5.0	-5.00011	-5.00016	-5.02878	-2.4993
11	8.0	8.00008	8.00007	8.01252	7.19414
12	0.3	0.299975	0.299993	0.296701	0.445809

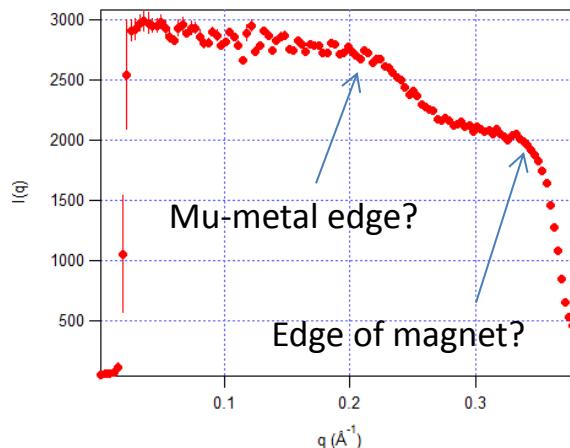
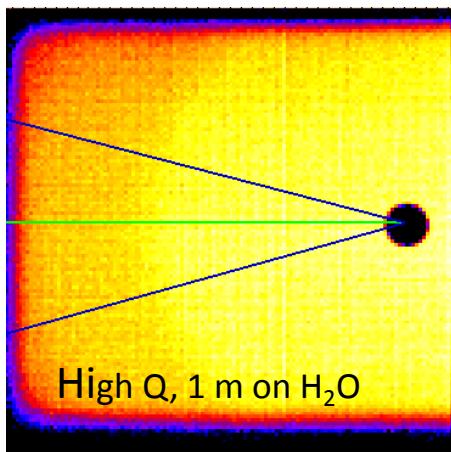


Rule of minimum q needed $\sim 2\pi/\text{feature size}$
 Example: $\pi/(2 \cdot \text{radius}) = 2\pi/20 \text{ \AA} = 0.31 \text{ \AA}^{-1}$

Experimental $0.18 \text{ \AA}^{-1} \rightarrow \sim 35 \text{ \AA}$ minimum (18 \AA inner radius)

If signal drops $\sim 0.12 \text{ \AA}^{-1} \rightarrow \sim 50 \text{ \AA}$ minimum (25 \AA radius) ?

1D Case: What resolution is appropriate for our experiment?



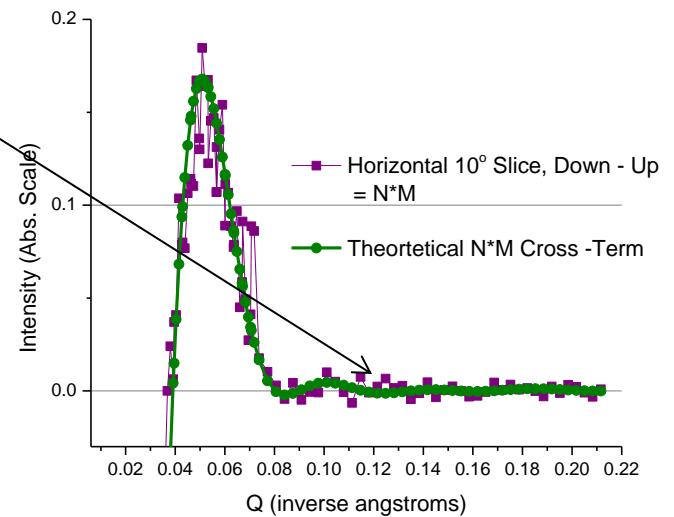
SASCALC predicts 0.44 \AA^{-1} without magnet at Huber position and 0.64 \AA^{-1} at sample chamber. However, we also want a moderate field of about 0.5 T.

Experimental $0.2 \text{ \AA}^{-1} \rightarrow \sim 31 \text{ \AA}$ minimum (16 \AA inner radius)

If signal drops $\sim 0.12 \text{ \AA}^{-1} \rightarrow \sim 50 \text{ \AA}$ minimum (25 \AA inner radius)?

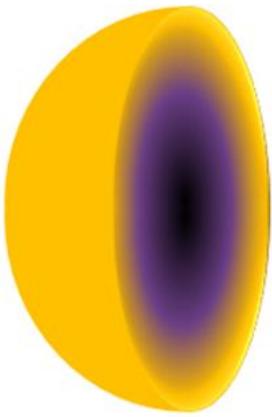
A reasonable range might be for Q up to 0.12 \AA^{-1} :

Radii(Å)	Input Rho	Inverted Rho
30	0.5	0.506635
40	-2.0	-2.0186
50	1.0	1.01321
75	-3.0	-3.00165
100	5.0	5.00067
110	1.5	1.49926
150	0.1	0.10003



1D Case: Simulating Experiment (check of magnetic core and diameter)

Simulate Nucl-Mag C.T. and Mag. Ampl.:

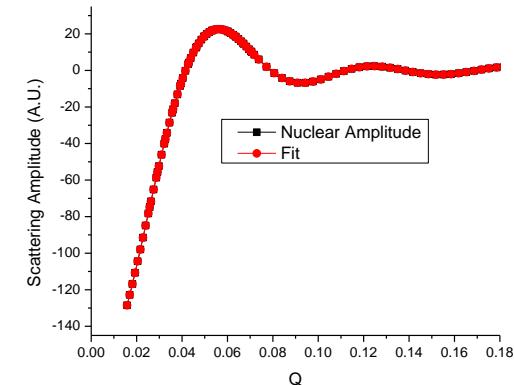
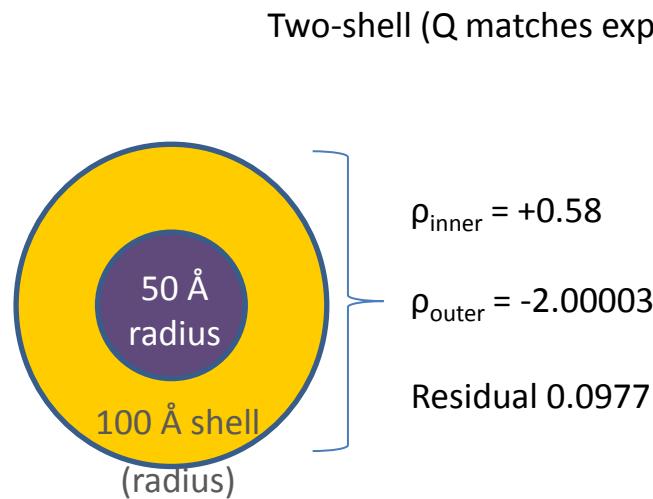


10 nm diameter Fe_3O_4
MSLD = $1.46 \times 10^{-6} \text{ \AA}^{-1}$

10 nm diameter Fe_3O_4
 $\Delta \text{SLD} = 0.58 \times 10^{-6} \text{ \AA}^{-1}$

5 nm thick Protein G
 $\Delta \text{SLD} = -2.0 \times 10^{-6} \text{ \AA}^{-1}$

D_2O Buffer
 $\Delta \text{SLD} = 0.0 \times 10^{-6} \text{ \AA}^{-1}$



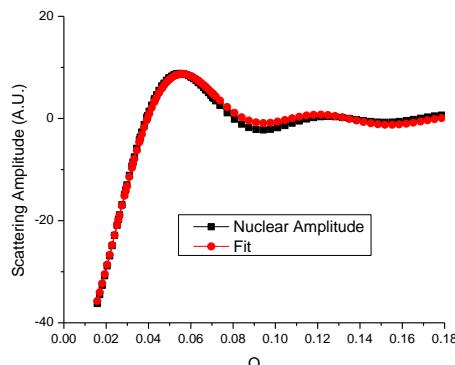
Input (alter internal boundary):

Mag. Radius = 50 Å

Nucl. Core = 40 Å radius $\rightarrow \rho = 0.58$

Nucl. Shell = 100 Å radius $\rightarrow \rho = -0.592$

Residual = 73



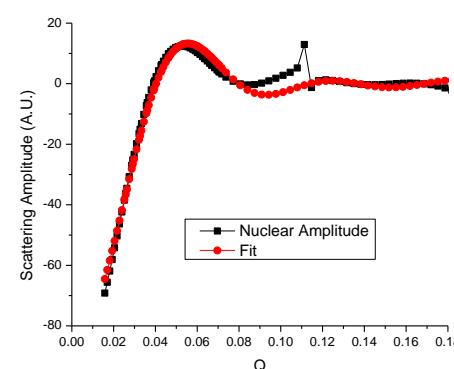
Input (shrink mag. core):

Mag. Radius = 40 Å

Nucl. Core = 50 Å radius $\rightarrow \rho = 0.58$

Nucl. Shell = 100 Å radius $\rightarrow \rho = -39.48$

Residual = 273



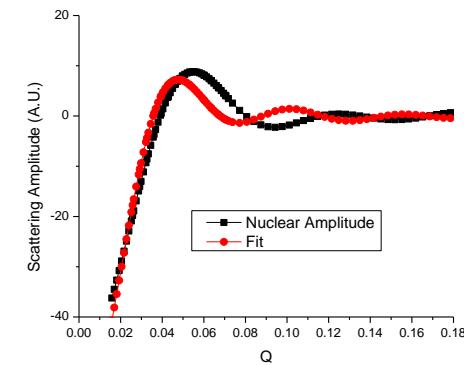
Input (increase total size):

Mag. Radius = 50 Å

Nucl. Core = 40 Å radius $\rightarrow \rho = 0.58$

Nucl. Shell = 120 Å radius $\rightarrow \rho = -4.304$

Residual = 340



1D Case: Inversion in Spherical Coordinates (Data!)

Can be fairly certain that total nuclear radius is ~9-10 nm; mag. core radius is ~50 Å (company specs)

Q of SANS 0.01579 Å⁻¹ to 0.18 Å⁻¹

Trial Input (three quadrants):

Mag. Radius = 55 Å

Nucl. Core = 50 Å radius

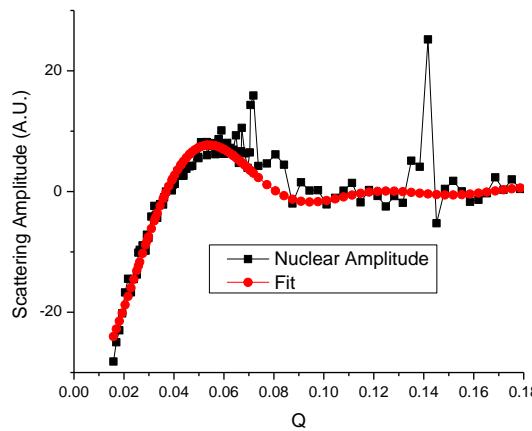
Nucl. Shell_1 = 100 Å radius

Nucl. Shell_2 = 150 Å radius



$$\rho_{\text{core}} = 0.58 \quad \rho_{\text{shell1}} = -0.728 \\ \rho_{\text{shell2}} = 0.00265$$

Residual 299



Trial Input (three quadrants):

Mag. Radius = 50 Å

Nucl. Core = 50 Å radius

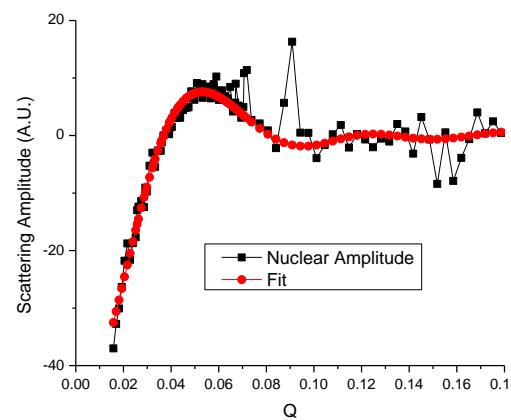
Nucl. Shell_1 = 100 Å radius

Nucl. Shell_2 = 150 Å radius



$$\rho_{\text{core}} = 0.58 \quad \rho_{\text{shell1}} = -1.16549 \\ \rho_{\text{shell2}} = -0.069641$$

Residual 280



Trial Input (three quadrants):

Mag. Radius = 45 Å

Nucl. Core = 50 Å radius

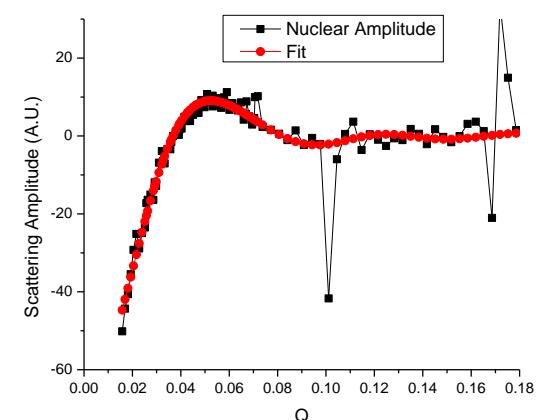
Nucl. Shell_1 = 100 Å radius

Nucl. Shell_2 = 150 Å radius



$$\rho_{\text{core}} = 0.58 \quad \rho_{\text{shell1}} = -1.58535 \\ \rho_{\text{shell2}} = -0.14284$$

Residual 435



Conclusion: Magnetic core 40 to 50 angstroms in radius.

IPG Core-Shell Inversion

Mag. Core = 50 Å radius

Radius 30 Å: $\rho = 0.316$

Radius 40 Å: $\rho = 0.580$

Radius 50 Å: $\rho = -0.431$

Radius 75 Å: $\rho = -0.230$

Radius 100 Å: $\rho = -0.127$

Radius 110 Å: $\rho = -0.022$

Radius 150 Å: $\rho = -0.022$

Mag. Core = 40 Å radius

Radius 30 Å: $\rho = -0.460$

Radius 40 Å: $\rho = 0.580$

Radius 50 Å: $\rho = 0.292$

Radius 75 Å: $\rho = -0.380$

Radius 100 Å: $\rho = -0.107$

Radius 110 Å: $\rho = -0.044$

Radius 150 Å: $\rho = -0.034$

“Core”

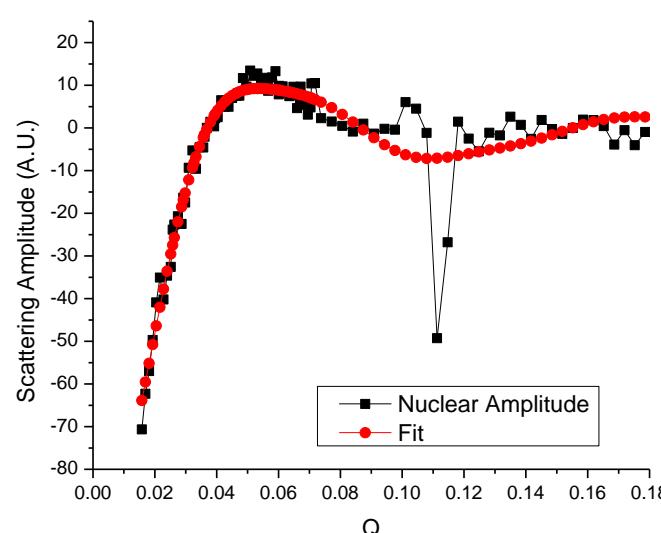
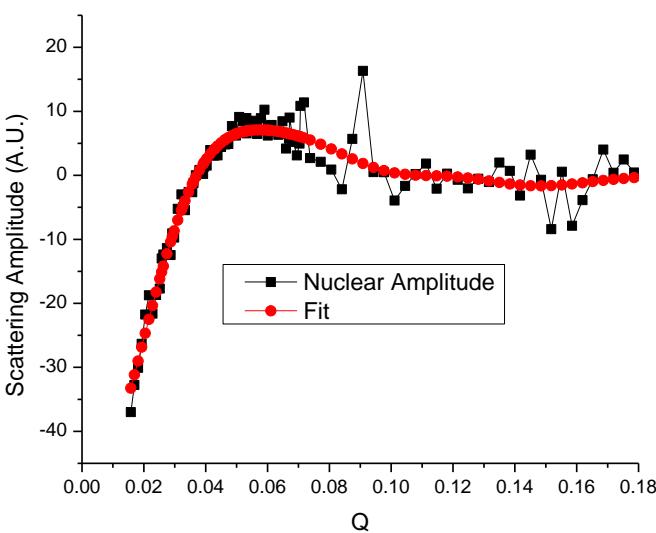
“Core”

“Shell”

“Shell”

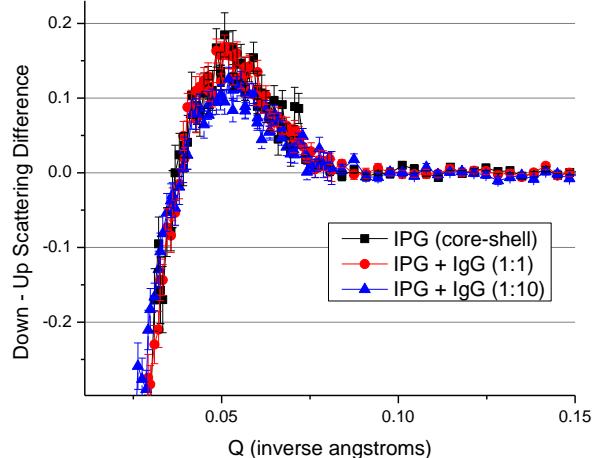
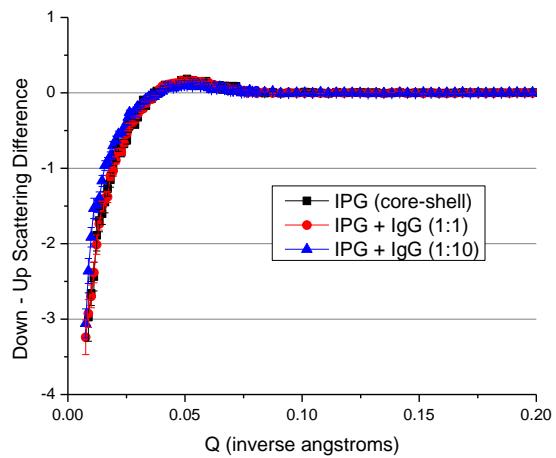
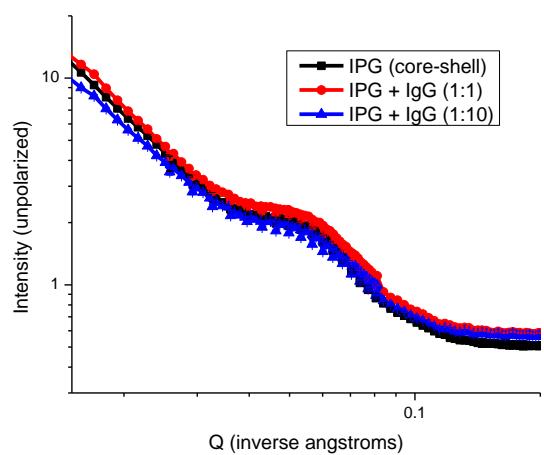
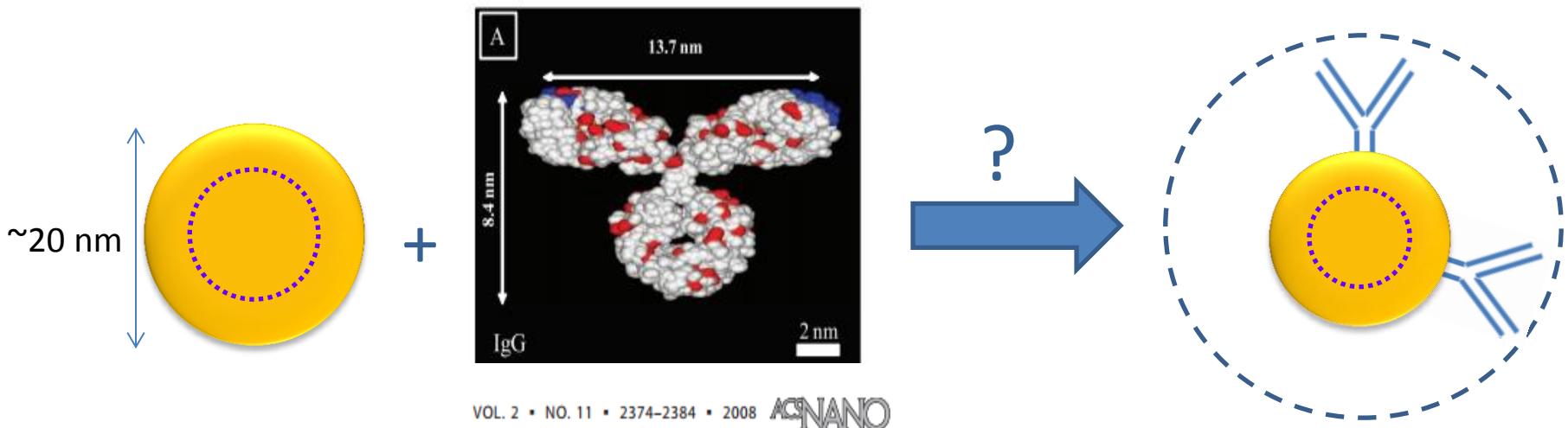
“Solvent”

“Solvent”



At limit of 40 to 50 Å radius magnetic core. Do see + core, -protein shell, buffer at 100 Å and beyond.

Did we tag IPG core-shell with IgG anti-bodies?



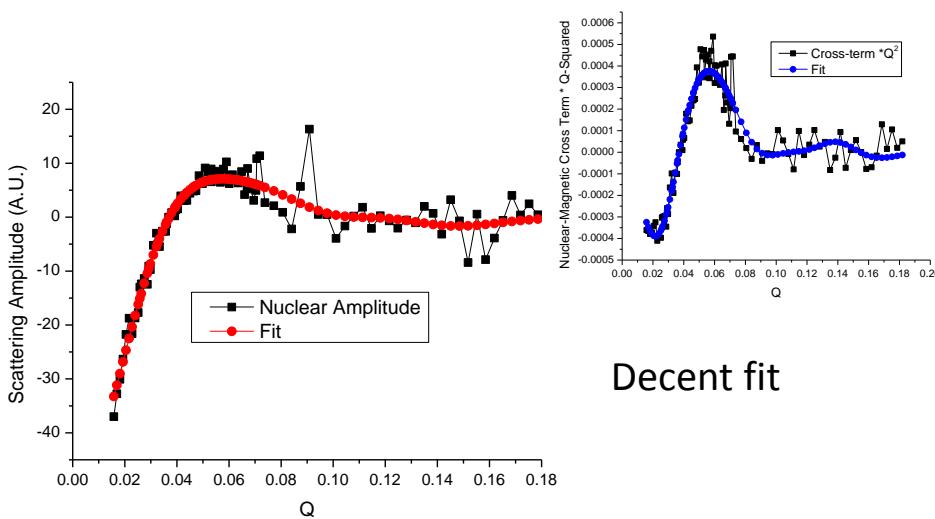
Similar zero-crossings of Nucl-Mag C.T. indicate virtually same-sized particles.

IPG + IgG Inversions

Mag. Core = 50 Å radius

IPG Alone

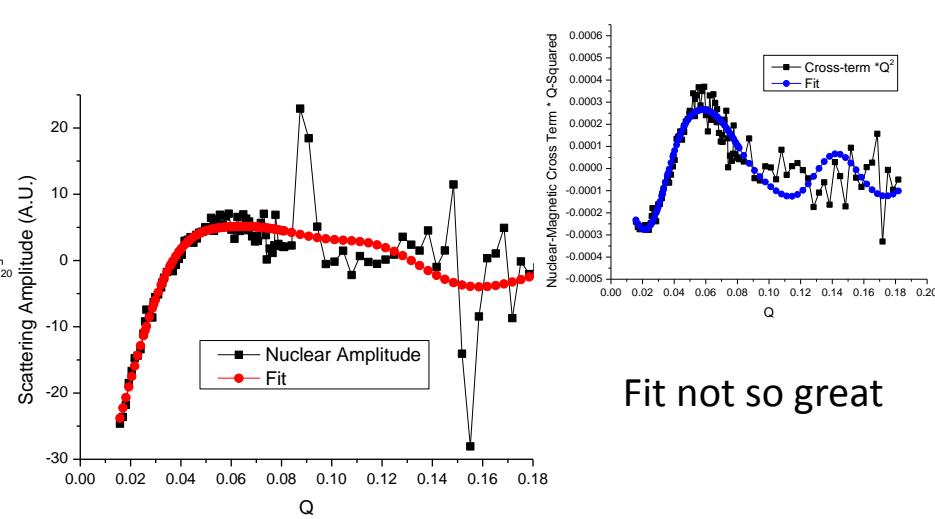
Radius 30 Å:	$\rho = 0.316$	"Core"
Radius 40 Å:	$\rho = 0.580$	
Radius 50 Å:	$\rho = -0.431$	
Radius 75 Å:	$\rho = -0.230$	"Protein Shell"
Radius 100 Å:	$\rho = -0.127$	
Radius 110 Å:	$\rho = -0.022$	
Radius 150 Å:	$\rho = -0.022$	"Solvent"



Decent fit

IPG + 10 x's IgG

Radius 30 Å:	$\rho = 0.194$	"Core"
Radius 40 Å:	$\rho = 0.580$	
Radius 50 Å:	$\rho = -0.670$	"Protein Shell"
Radius 75 Å:	$\rho = -0.006$	
Radius 100 Å:	$\rho = -0.060$	"Solvent"
Radius 110 Å:	$\rho = 0.0031$	
Radius 150 Å:	$\rho = -0.0087$	



Fit not so great

Conclusion: Data of IPG + IgG is not of sufficient quality (several hours) to distinguish from IPG only.

Recap on Spherical Inversion

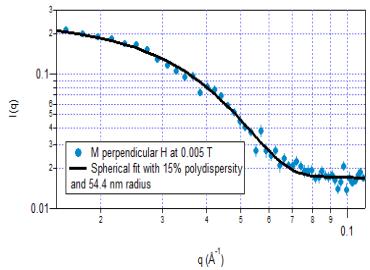
- Using Nuclear-Magnetic cross-term adds additional information to conventional SANS

Pros:

~10% polydispersity in spherical magnetic reference OK
 Can characterize magnetic reference from PASANS →
 $\downarrow - \uparrow$ difference eliminates background scattering!
 Partial polarization or partial magnetic saturation OK

Cons:

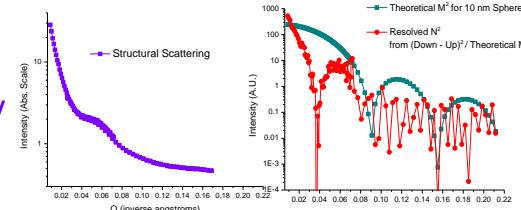
Need magnetic reference shape
 ~50 mg magnetic tags for PASANS



- Point-by-point $[N^2 \propto (\downarrow - \uparrow)^2 / (\text{known } M \text{ scattering})]$

Easy to implement on any Q-range
 Enhanced sensitivity to outer N^2 boundary

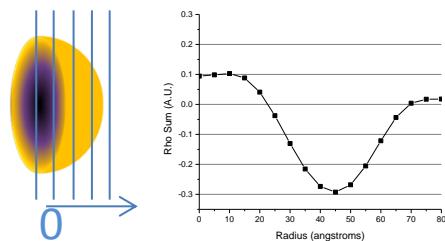
Limited internal sensitivity



- Inverse Fourier Transform (cosine series)

Cross-sectional SLD provides internal sensitivity

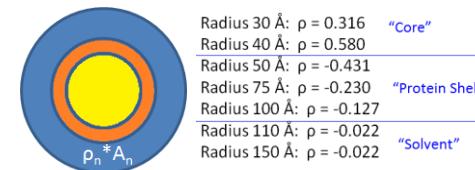
Med-large Q-range
 Need low statistical unc.
 Prone to ringing



- Full SLD Inversion (over-determined set of equations, Gauss-Jordan elimination or single-value decomposition)

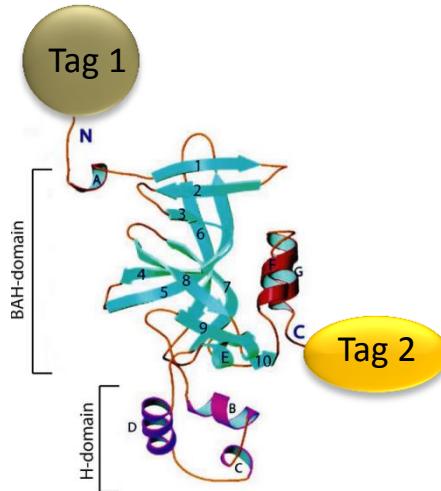
Maximum internal sensitivity
 Q-range of $0.3 \text{ Å}^{-1} \rightarrow 1 \text{ nm resolution}$
 3D analysis possible with appropriate tags

Sensitive to box-size binning
 Need low statistical unc.

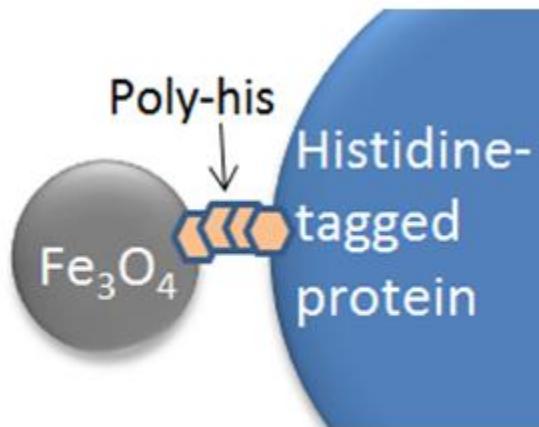


- What other core-shell systems could benefit from this type of analysis (Ca in casein, polymer, surfactant thickness)?

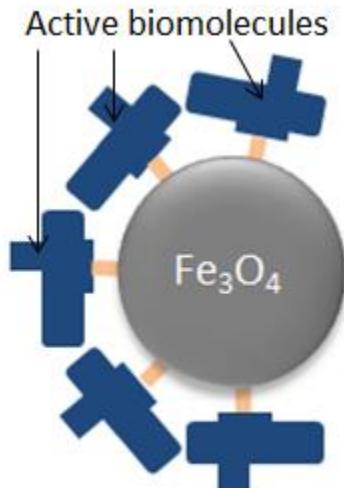
Anisotropic Tagging Ideas



Multiple-site tags



Site-specific Tagging



Many sites / tag

(Image from dissertation of Antje Geißenhöner, The role of the N-terminal acetyltransferase NatA in transcriptional silencing in *Saccharomyces cerevisiae*)

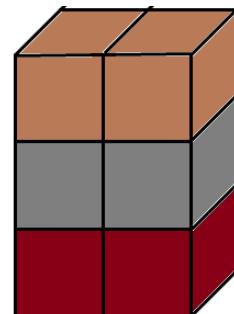


Micelle or other
object of interest

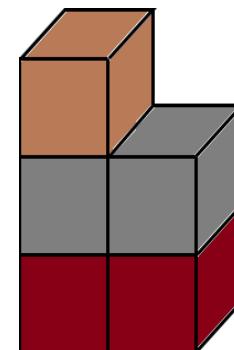
Magnetic wire with
functionalized end(s)

Nanowire Tagging

Anisotropic tags:

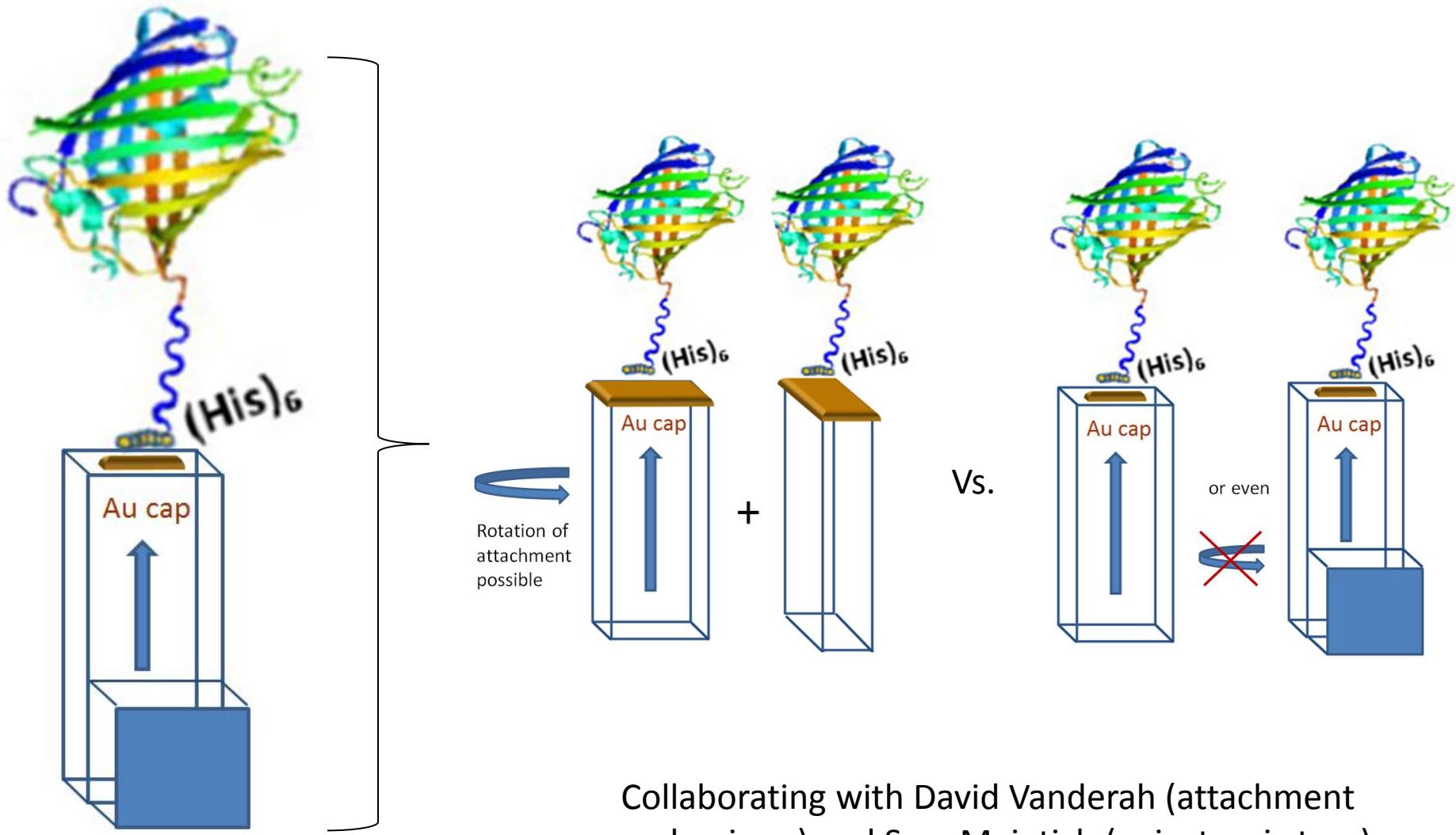


Rectangular tags



L-shaped tags

Idea for Oriented Anisotropic Tag Attachment



Collaborating with David Vanderah (attachment mechanisms) and Sara Majetich (anisotropic tags)

Thank you for your attention!

Questions?