β-Decay and the Precision Frontier

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Lecture 1: on the symmetries of the Standard Model (SM) and the role β-decay played in their elucidation

Lecture 2: on β-decay and precision tests of the SM at the quantum level

Lecture 3: on difficulties with the SM and how neutron observables open windows to their resolution

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The Standard Model

describes all known electromagnetic, weak, and strong interaction phenomena in a formalism with predictive power.

It has gauge bosons γ, $W^{\pm}$, Z^0, g and three generations of quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} c \\ s \end{pmatrix} \qquad \begin{pmatrix} t \\ b \end{pmatrix}$$

and leptons

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \qquad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \qquad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

and a fundamental scalar $H_{\rm SM}$ which has not yet been found.

on Mass

Mass is key to explaining the relative strength of the weak and electromagnetic interactions.

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

What is the origin of mass?

The gauge boson masses, as well as the masses of the elementary fermions, arise through the spontaneously breaking of a local gauge symmetry.
This is known as the "Higgs mechanism".

The Higgs mechanism is also key to describing the quark mixing observed under the weak interactions: it gives rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Spontaneous Symmetry Breaking

Let's begin with the example of a Heisenberg ferromagnet: $\mathcal{H} = -g \sum_{i,j} \mathcal{S}_i \cdot \mathcal{S}_j$ with $g > 0$.

At $T < T_c$ the system develops a non-zero magnetization $\mathcal{M} \neq 0$.

The Hamiltonian is rotationally invariant, but its ground state is not – the symmetry of $\mathcal{H}$ is hidden.

Spontaneous symmetry breaking (SSB) also operates in QCD.

The u, d quarks are *very* light compared to M_p. If $m_q = 0$

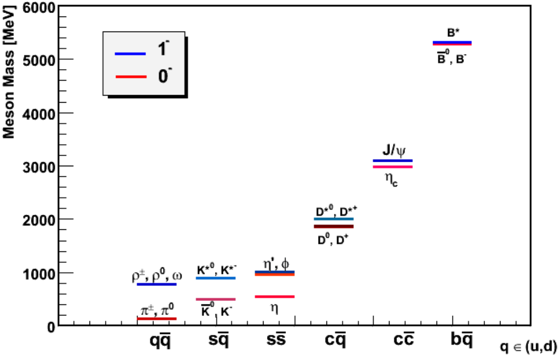
$$\mathcal{L}_{\rm QCD} = \mathcal{L}^L_{\rm QCD} + \mathcal{L}^R_{\rm QCD}$$

If this chiral symmetry were explicit, one would expect the low-lying hadronic spectrum to contain parity doublets, but this does not occur.

Perhaps the axial vector currents are *spontaneously broken*. [Nambu and Jona-Lasinio, Phys. Rev. 122, 345 (1961).]

Goldstone's Theorem: For every spontaneously broken continuous symmetry, the theory must contain a massless particle (Goldstone boson). [Goldstone, Nuovo Cim. 19, 154 (1961)]

The Pattern of Low-Lying Meson Masses

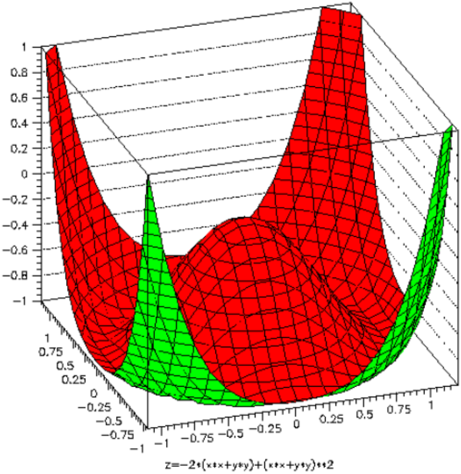


- Masses of states which differ only in (u, d) are nearly degenerate.
- There are eight low-lying 0^- states — $\pi^\pm, \pi^0, K^0, \bar{K}^0, K^\pm$, and η — the η' is much heavier.

We can explain this pattern by invoking symmetries which are, in turn, approximate (isospin), spontaneously broken (chiral), and anomalous (axial $U(1)$).

Spontaneous Symmetry Breaking

Here's a class of potentials which can be used to describe the spontaneous breaking of a continuous symmetry...



A "Mexican Hat" Potential

Secret Symmetry

Let's see how this can work. Consider the potential for a real field ϕ. Suppose $\mu^2 > 0$, real and $\lambda > 0$.

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

This is symmetric under $\phi \to -\phi$, and the minimum energy state is $\phi = 0$.
What if $\mu^2 \to -\mu^2$? Then

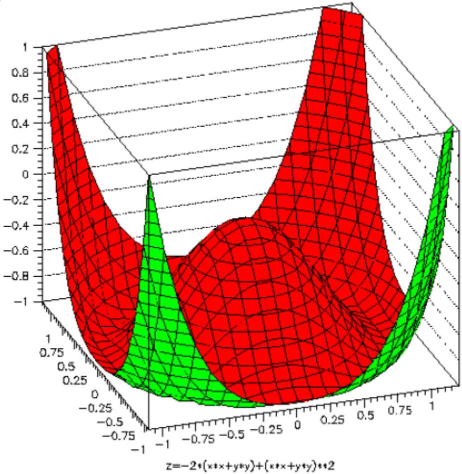
$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

Now the minimum energy state corresponds to $\phi \neq 0$! There are two minima.
Expanding ϕ about one minimum, $\phi(x) = v + \sigma(x)$, e.g., we would find that the new potential no longer had $\sigma \to -\sigma$ manifest.
If we had looked at the potential corresponding to the surface of revolution of this potential ($N = 2$ scalar fields), we would have had a continuous symmetry, and if we had expanded about the vacuum expectation value, we would have found a massless state.

The Higgs Mechanism

A continuous, local symmetry can be spontaneously broken without yielding Goldstone bosons; rather, the gauge bosons gain mass. Our by now-familar potential is that of the Higgs scalar field

For now, we set aside the question of why the $W^{\pm}$ and Z gauge bosons have the masses that they do; this mechanism provides no explanation for this, nor for the pattern of fermion masses.



A "Mexican Hat" Potential

Spontaneous Symmetry Breaking without Goldstone Bosons

Here we work in the context of a theory with local gauge invariance – and that makes all the difference!

Let's consider QED with no fermions but with a complex scalar field:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)(D^\mu\phi^*) - V(\phi)$$

This $\mathcal{L}$ is invariant under

$$\phi(x) \to e^{i\alpha(x)}\phi(x) \quad ; \quad A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

This is a $U(1)$ symmetry.

N.B. ϕ is coupled to the photon through $D_\mu = \partial_\mu - ieA_\mu$.

Now when we expand ϕ about the minimum of $V(\phi)$, spontaneously breaking the local gauge symmetry, we find that our would-be massless state gives mass to the photon!

A model for the Meißner effect in Type I superconductors! [Landau-Ginzburg theory!]

This mechanism generalizes to non-Abelian gauge theory.

The Glashow-Salam-Weinberg Model

To build the Standard Model, we must

- Pick the gauge group: The electroweak portion of the Standard Model is a quantum field theory based on a $SU(2)_L \times U(1)_Y$ local gauge symmetry.
- Choose the particle content and its group representations and charge assignments:

  We put a complex scalar field in a $SU(2)_L$ doublet. This upon SSB will yield 3 massive gauge bosons: $W^\pm, Z^0$. Since the $W^\pm$ carry electric charge, electromagnetism must "lie across" $SU(2)_L \times U(1)_Y$. N.B. $Q = T_3 + Y$. Note ϕ has $Y = +1/2$.

  We put the fermions in left-handed doublets and right-handed singlets generation by generation.

  E.g.,

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \equiv E_L \qquad T_3 = \pm\frac{1}{2} \;\; Y = -\frac{1}{2}$$

$$e_R \qquad T_3 = 0 \;\; Y = -1$$

Fermion Masses

We cannot give the fermions mass as in QED because

$$m_e \bar{\psi}\psi = m_e(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

is forbidden by SU(2)$_L \times$U(1) gauge invariance!
We must use the Higgs mechanism!

$$\mathcal{L}_{e\,\mathrm{mass}} = -\lambda_e \bar{E}_L \phi e_R + h.c.$$

with $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}_{e\,\mathrm{mass}} = -\frac{1}{\sqrt{2}}\lambda_e v \bar{e}_L e_R + h.c.$$

and we identify the electron mass $m_e = \frac{1}{\sqrt{2}}\lambda_e v$.
For 3 generations of down-like and up-like quarks:

$$\mathcal{L}_{q\,\mathrm{mass}} = -\lambda_d^{ij} \bar{Q}_L^i \phi d_R^j - \lambda_u^{ij} \bar{Q}_L^i \phi^\dagger u_R^j + h.c.$$

CP is broken if $\lambda_{d,u}^{ij}$ are complex!
N.B. Q_L has $T_3 = \pm 1/2$, $Y = +1/6$; u_R has $Y = +2/3$; d_R has $Y = -1/3$

Quark Masses and Mixings

The $\lambda_{d,u}^{ij}$ can be anything! They are only constrained by symmetry before we fit them to experiment.

We can simplify by rotating to a basis in which the quark mass matrix is diagonal

$$u_L'^{\,i} = \mathcal{U}_u^{ij} u_L^j \quad ; \quad d_L'^{\,i} = \mathcal{U}_d^{ij} d_L^j$$

This complicates the expression, however, for the quark charged weak current:

$$\begin{aligned} J^\mu_{W^+} &= \frac{1}{\sqrt{2}} \bar{u}_L^i \gamma^\mu d_L^i \\ &= \frac{1}{\sqrt{2}} \bar{u}_L'^{\,i} \gamma^\mu (\mathcal{U}_u^\dagger \mathcal{U}_d)_{ij} d_L'^{\,i} \end{aligned}$$

Enter the Cabibbo-Kobayashi-Maskawa matrix:

$$\boxed{V_{\mathrm{CKM}} = \mathcal{U}_u^\dagger \mathcal{U}_d}$$

$\mathcal{U}_{u,d}$ is clearly unitary, and thus V_{CKM} is also.

The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

The decay $K^- \to \mu^- \bar{\nu}_\mu$ occurs: the quark mass eigenstates *mix* under the weak interactions. By convention

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_{\rm weak} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm mass} \quad ; \quad V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In the Wolfenstein parametrization (1983)

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

where $\lambda \equiv |V_{us}| \simeq 0.22$ and is thus “small”. A, ρ, η are real.

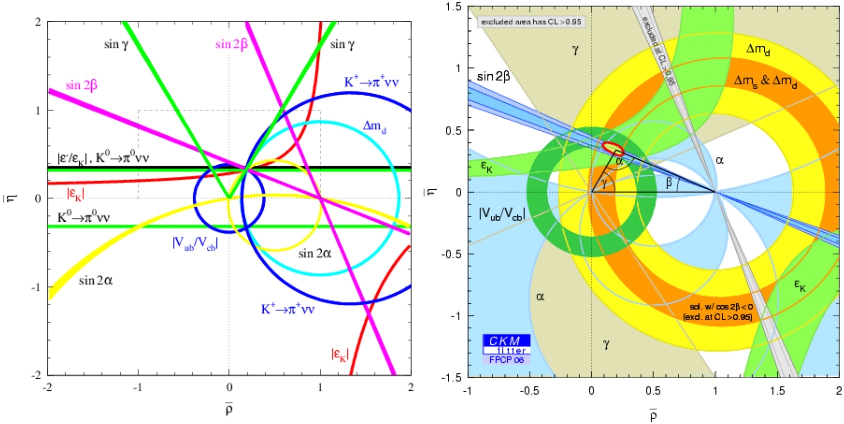
Testing the Quark Mixing Matrix

In the Standard Model (SM)

- There are three "generations" of particles. Thus, the CKM matrix is unitary.
- The unitarity of the CKM matrix and the structure of the weak currents implies that four parameters capture the CKM matrix.
- A real, orthogonal 3×3 matrix is captured by three parameters. The fourth parameter (η) must make $V_{\rm CKM}$ complex.
- All CP-violating phenomena are encoded in η.

> To test the SM picture of CP violation we must test the relationships it entails.

Testing CKM Unitarity – "the" Unitarity Triangle



[CKMfitter: hep-ph/0104062, hep-ph/0406184 ; http://ckmfitter.in2p3.fr – April, 2006 update]

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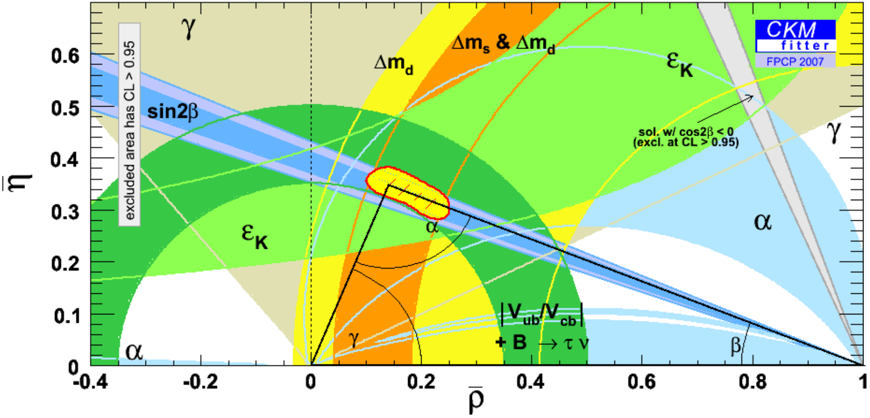


[CKMfitter: hep-ph/0104062, hep-ph/0406184 ; http://ckmfitter.in2p3.fr – June, 2007 update]

The possibility of non-SM CP violation is gradually being relegated to a smaller and smaller role, but some intriguing discrepancies remain....

Using the global fits...

The first row of the CKM matrix yields the most precise test of CKM unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992 \pm 0.0011$$

whereas

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1.001 \pm 0.005$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.968 \pm 0.181$$

N.B. the W leptonic width and the V_{ud} unitarity test (1^{st} row) yields

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.003 \pm 0.027$$

cf. $\alpha + \beta + \gamma = 184^{+20}_{-15}{}^{\circ}$.

["The CKM Quark-Mixing Matrix," PDG, 2006.]

Tests to this precision require precise computations of SM radiative corrections...

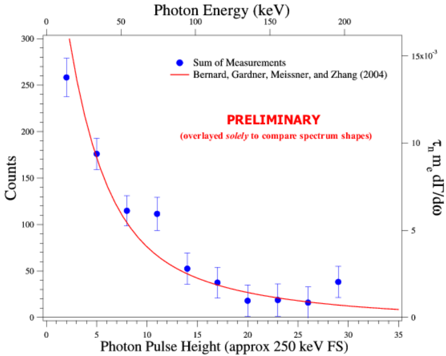
$$g_v = V_{ud}(1 + \Delta\hat{r}_\beta - \Delta\hat{r}_\mu)$$

The precision of the current unitarity tests probes the reliability of these computations to the sub-1% level.

Testing SM radiative corrections

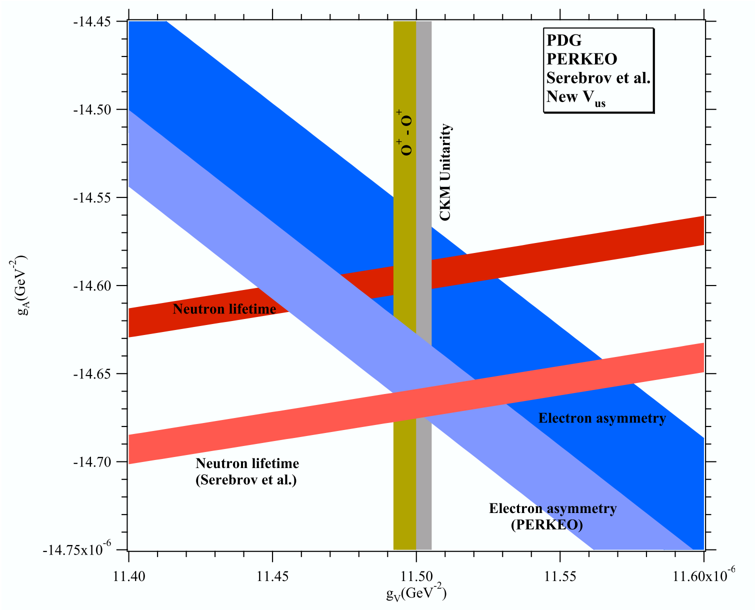
SM radiative corrections relate the vector weak coupling constant g_V of the nucleon to V_{ud}.

The decay $n \to p e^- \bar{\nu}_e \gamma$ has finally been observed.



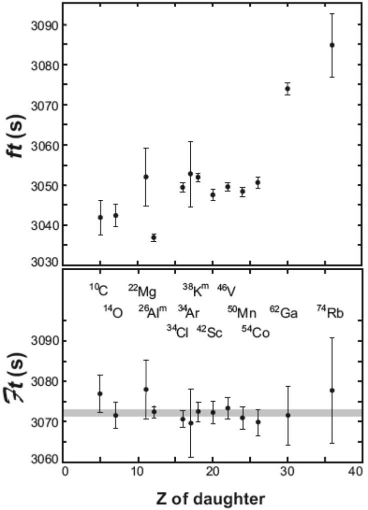
For $\omega \in [0.015\,\text{MeV}, 0.340\,\text{MeV}]$ we find a Br of $2.85 \cdot 10^{-3}$, cf. with the expt'l result of $3.13 \pm 0.34 \cdot 10^{-3}$. [Nico et al. (NIST), Nature, 2006]

The $\mathcal{O}(1/M)$ terms contribute $\mathcal{O}(0.04\%)$ to the Br.


PDG
PERKEO
Serebrov et al.
New V_{us}
$0^+ - 0^+$
CKM Unitarity
Neutron lifetime
Neutron lifetime
(Serebrov et al.)
Electron asymmetry
Electron asymmetry
(PERKEO)
-14.45
-14.50
-14.55
-14.60
-14.65
-14.70
-14.75x10^{-6}
g_A(GeV^{-2})
11.40
11.45
11.50
11.55
11.60x10^{-6}
g_V(GeV^{-2})

[J. Nico (NIST)]

The best determination of V_{ud} comes from $0^+ \to 0^+$ decays in nuclei. [Hardy and Towner, arXiv:0812.1202v1]



The corrected $\mathcal{F}t$ values require nuclear-structure-dependent corrections:

$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1+\Delta_R^V)}$

Note i) nuclear axial radiative correction, ii) charge-dependent nuclear matrix element overlap, iii) hadronic structure in the $\gamma W^{\pm}$ box. ii) has been recently criticized [Miller and Schwenk, 2008]

Note breaking developments vis-a-vis iii) [Gorchtein and Horowitz, 2008]

Bibliography

The following textbooks provide helpful background and should aid in assimilating the information found in the journal articles cited throughout.

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