β-Decay and the Rise of the Standard Model

Susan Gardner

Fermi National Accelerator Laboratory
Batavia, IL 60510 USA

Department of Physics and Astronomy*
University of Kentucky
Lexington, KY 40506 USA

gardner@pa.uky.edu

* Permanent Address.

Lecture 1: on the symmetries of the Standard Model (SM) and the role β-decay played in their elucidation

Lecture 2: on β-decay and precision tests of the SM at the quantum level

Lecture 3: on difficulties with the SM and how neutron observables open windows to their resolution

UNIVERSITY OF KENTUCKY

Inspiration

"It is a part of the adventure of science to try to find a limitation in all directions and to stretch a human imagination as far as possible everywhere. Although at every stage it has looked as if such an activity was absurd and useless, it often turns out at least not to be useless."
Richard P. Feynman,
in "Computing Machines of the Future",
from Feynman and Computation, A. J. G. Hey, ed., 2002

How "Weak" is the Weak Interaction?

We know of four fundamental interactions: electromagnetic, strong, weak, and gravitational.

Let's set gravity aside and consider the others exclusively.

Particles of comparable mass can have very different lifetimes.

$$\pi^+ \to \mu^+ \nu_\mu \qquad [99.98\% \text{ of all } \pi^+ \text{ decays}]\,; \qquad \tau_{\pi^+} \sim 2.6 \cdot 10^{-8}\,\text{s}$$
$$\pi^0 \to 2\gamma \qquad [98.8\% \text{ of all } \pi^0 \text{ decays}]\,; \qquad \tau_{\pi^0} \sim 8.4 \cdot 10^{-17}\,\text{s}.$$

$$\Gamma \propto \tau^{-1} \Longrightarrow \frac{|g_{\rm eff}^{em}|^2}{|g_{\rm eff}^{weak}|^2} \sim 10^8 \Longrightarrow |g_{\rm eff}^{em}| \sim 10^4 |g_{\rm eff}^{weak}|$$

whereas

$$\rho^0 \to \pi^+ \pi^- \qquad [\sim 100\% \text{ of all } \rho^0 \text{ decays}]$$
$$\rho^0 \to \mu^+ \mu^- \qquad [\sim 4.6 \cdot 10^{-5} \text{ of all } \rho^0 \text{ decays}]$$

$$\Longrightarrow \frac{|g_{\rm eff}^{em}|^2}{|g_{\rm eff}^{str}|^2} \sim 4 \cdot 10^{-5} \Longrightarrow |g_{\rm eff}^{str}| \sim 10^2 |g_{\rm eff}^{em}|$$

Conclude weak interaction is $\sim 10^6$ times weaker than the strong interaction!

The Standard Model

describes all known electromagnetic, weak, and strong interaction phenomena in a formalism with predictive power.

It has gauge bosons $\gamma, W^{\pm}, Z^0, g$ and three generations of quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

and leptons

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

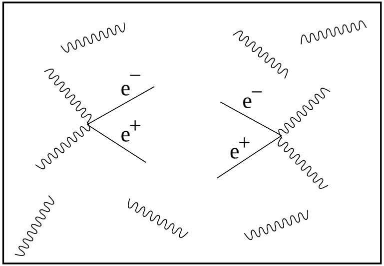
and a fundamental scalar $H_{\rm SM}$ which has not yet been found.

The Discrete Symmetries – C, P, and T

In particle interactions, can we tell...

- Left from Right? (P)
- Positive Charge from Negative Charge? (C)
- Forward in Time from Backward in Time? (T)
- Matter from Antimatter? (CP)

If we “observed” a box of photons at constant temperature $T \sim m_e$, interacting via electromagnetic forces, the answer would be No.



However, ...

On the Possibility of Parity Violation

Context: Dirac – the existence of a magnetic monopole can explain the quantization of electric charge! [Dirac, Proc. Roy. Soc. London A 133, 60 (1931)]

$$\nabla \cdot E = 4\pi\rho \quad ; \quad \nabla \cdot B = 0 \Longrightarrow 4\pi\rho_M$$

Dirac also showed that the circulation of opposite magnetic monopoles in the nucleon could give rise to a nonzero *electric dipole moment*.

[Dirac, Phys. Rev. 74, 817 (1948).]

The electric dipole moment d of a nonrelativistic particle with spin S is defined via $\mathcal{H} = -d\frac{\mathbf{S}}{S} \cdot \mathbf{E}$

But both quantities violate P and T!

E. M. Purcell and N. F. Ramsey, "On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei," Phys. Rev. 78, 807 (1950):

> The argument against electric dipoles, in another form, raises the question of parity.... But there is no compelling reason for excluding this possibility....

Discrete Symmetries — P, T, and C

Parity P:

Parity reverses the momentum of a particle without flipping its spin.

$$P a_p^s P^\dagger = a_{-p}^s \quad , \quad P b_p^s P^\dagger = -b_{-p}^s \qquad \Longrightarrow P\psi(t,x)P^\dagger = \gamma^0\psi(t,-x)$$

Time-Reversal T:

Time-reversal reverses the momentum of a particle and flips its spin. It is also antiunitary; note $[x,p] = i\hbar$.

$$T a_p^s T^\dagger = a_{-p}^{-s} \qquad T b_p^s T^\dagger = b_{-p}^{-s} \qquad \Longrightarrow T\psi(t,x)T^\dagger = -\gamma^1\gamma^3\psi(-t,x)$$

Charge-Conjugation C:

Charge conjugation converts a fermion with a given spin into an antifermion with the same spin.

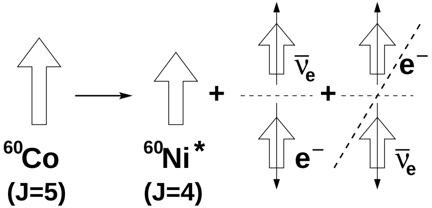
$$C a_p^s C^\dagger = b_p^s \quad , \quad C b_p^s C^\dagger = a_p^s \qquad \Longrightarrow C\psi(t,x)C^\dagger = -i\gamma^2\psi^*(t,x)$$

The Weak Interactions Violate Parity

There is a "fore-aft" asymmetry in the e^- intensity in $^{60}\vec{Co}$ β-decay....
[Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957);

note also Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415 (1957); http://focus.aps.org/story/v22/st19 .]

Schematically



$$I_e(\theta) = 1 - \frac{\vec{J}\cdot\vec{p}_e}{E_e}$$

P is violated in the weak interactions!

Both P and C are violated "maximally"

$$\Gamma(\pi^+ \to \mu^+\nu_L) \neq \Gamma(\pi^+ \to \mu^+\nu_R) = 0 \quad ; \text{ P violation}$$

$$\Gamma(\pi^+ \to \mu^+\nu_L) \neq \Gamma(\pi^- \to \mu^-\overline{\nu}_L) = 0 \quad ; \text{ C violation}$$

The "Two-Component" Neutrino

A Dirac spinor can be formed from two 2-dimensional representations:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

In the Weyl representation for γ^μ,

$$(i\gamma^\mu \partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_0 + \sigma \cdot \nabla) \\ i(\partial_0 - \sigma \cdot \nabla) & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

If m=0, ψ_L and ψ_R decouple and are of definite helicity for all *p*.
Thus, e.g.,

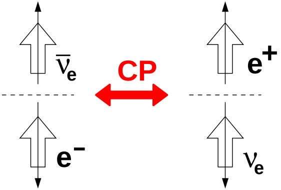
$$i(\partial_0 - \sigma \cdot \nabla)\psi_L(x) \Longrightarrow E\psi_L = -\sigma \cdot p\,\psi_L$$

$$\sigma \cdot \hat{p}\,\psi_L = -\psi_L$$

Note $\bar{\psi} \equiv \psi_L^\dagger \gamma^0$ transforms as a right-handed field.
Experiments $\Longrightarrow$ No "mirror image states": neither $\overline{\nu}_L$ nor ν_R exist.
Possible only if the neutrino is of zero mass.

The Weak Interactions Can Also Violate CP

CP could be a good symmetry even if P and C were violated.
Schematically



$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \overline{\nu}_R) \quad ; \text{ CP invariance!}$$

Weak decays into hadrons, though, can violate *CP*.
There are "short-lived" and "long-lived" K states:

$$K_S \sim \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0) \to \pi^+\pi^- \quad \text{(CP even)}$$

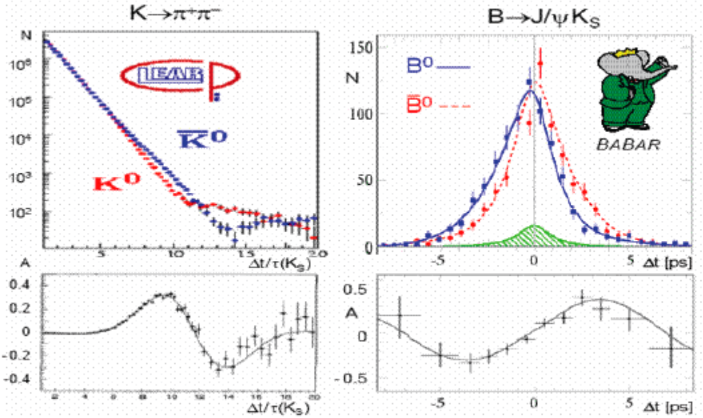
$$K_L \sim \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \to \pi^+\pi^-\pi^0 \quad \text{(CP odd)}$$

However, $K_L \to 2\pi$ as well! K_S and K_L do not have definite CP!

[Christenson, Cronin, Fitch, Turlay, PRL 13, 138 (1964).]

The decay rates for $K^0, \bar{K}^0 \to \pi^+\pi^-$ and $B^0, \bar{B}^0 \to J/\psi K_S$ are appreciably different.

[I.I. Bigi, arXiv:0703132v2 and references therein.]



All Observed Interactions Conserve CPT

The CPT Theorem

Any Lorentz-invariant, local quantum field theory in which the observables are represented by Hermitian operators must respect CPT. [Pauli, 1955; Lüders, 1954]

Coda: CPT violation implies Lorentz violation. [Greenberg, PRL 89, 231602 (2002)]

CPT $\Longrightarrow$ the lifetimes, masses, and the absolute values of the magnetic moments of particles and anti-particles are the same!

Note, e.g.,

$$\frac{|M_{K^0} - M_{\bar{K}_0}|}{M_{\mathrm{avg}}} < 10^{-18} \text{ @90\% } CL$$

$$\frac{|M_p - M_{\bar{p}}|}{M_{\mathrm{avg}}} < 10^{-8} \text{ @90\% } CL$$

Thus CP $\leftrightarrow$ T violation. Tests of CPT and Lorentz invariance are ongoing.

"A search for an annual variation of a daily sidereal modulation of the frequency difference between co-located ^{129}Xe and ^{3}He Zeeman masers sets a stringent limit on boost-dependent Lorentz and CPT violation involving the neutron, consistent with no effect at the level of 150 nHz...." [F. Canè et al., PRL 93 (2004) 230801]

Transformations of Lorentz Bilinears under P, T, and C

Notation: $\xi^\mu = 1$ for $\mu = 0$ and $\xi^\mu = -1$ for $\mu \neq 0$.
$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad ; \quad \sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

	$\bar\psi\psi$	$i\bar\psi\gamma_5\psi$	$\bar\psi\gamma^\mu\psi$	$\bar\psi\gamma^\mu\gamma_5\psi$	$\bar\psi\sigma^{\mu\nu}\psi$	∂_μ
	S	P	V	A	T	
P	$+1$	-1	ξ^μ	$-\xi^\mu$	$\xi^\mu\xi^\nu$	ξ^μ
T	$+1$	-1	ξ^μ	ξ^μ	$-\xi^\mu\xi^\nu$	$-\xi^\mu$
C	$+1$	$+1$	-1	$+1$	-1	$+1$
CPT	$+1$	$+1$	-1	-1	$+1$	-1

S is for Scalar
P is for Pseudoscalar
V is for Vector
A is for Axial-Vector
T is for Tensor

All scalar fermion bilinears are invariant under CPT.

Symmetries of a Dirac Theory

A Lagrangian must be a Lorentz scalar to guarantee Lorentz-invariant equations of motion. E.g., applying the Euler-Lagrange eqns to

$$\mathcal{L}_{\rm Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

yield Dirac equations for ψ and $\bar{\psi}$.
We can form two currents

$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x) \quad ; \quad j^{\mu 5}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$$

j^μ is always conserved if $\psi(x)$ satisfies the Dirac equation:

$$\partial_\mu j^\mu = (\partial_\mu\bar{\psi})\gamma^\mu\psi + \bar{\psi}\gamma^\mu\partial_\mu\psi = (im\bar{\psi})\psi + \bar{\psi}(-im\psi) = 0\,,$$

whereas $\partial_\mu j^{\mu 5} = 2im\bar{\psi}\gamma^5\psi$ — it is conserved only if $m = 0$.
By Noether's theorem a conserved current follows from an invariance in $\mathcal{L}_{Dirac}$:

$$\psi(x) \to e^{i\alpha}\psi(x) \quad ; \quad \psi(x) \to e^{i\alpha\gamma^5}\psi(x)$$

The last is a chiral invariance; it only emerges if $m = 0$.

Symmetries of a Dirac Theory

To understand why it is a chiral invariance, we note in the $m = 0$ limit that

$$j_L^\mu = \bar{\psi}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad j_R^\mu = \bar{\psi}\gamma^\mu \left(\frac{1+\gamma^5}{2}\right)\psi \,.$$

The vector currents of left- and right-handed particles are separately conserved.

Note in Weyl representation

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The factor $(1 \pm \gamma^5)$ acts to project out states of definite handedness.

$$\psi_L \equiv \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad \psi_R \equiv \left(\frac{1+\gamma^5}{2}\right)\psi \,.$$

so that $\mathcal{L} = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R = \mathcal{L}_L + \mathcal{L}_R$

Electromagnetism

We assert that if we couple a Dirac field ψ to an electromagnetic field A^μ j^μ is the electric current density. ψ can describe a free electron.

$$\psi(x)|p,s\rangle = u(p)e^{-ip\cdot x} \Longrightarrow (\gamma_\mu p^\mu - m)u(p) = 0\,.$$

By "canonical substitution" $p^\mu \to p^\mu + eA^\mu$

$$(\gamma_\mu p^\mu - m)u = \gamma^0 Vu \quad ; \quad \gamma^0 V = -e\gamma_\mu A^\mu$$

In $\mathcal{O}(e)$ the amplitude for an electron scattering from state $i \to f$ is

$$T_{fi} = -i\int u_f^\dagger V(x)u_i(x)\,d^4x = -i\int j_\mu^{fi}A^\mu\,d^4x \quad \text{with} \quad j_\mu^{fi} = -e\bar{u}_f\gamma_\mu u_i$$

For $e - p$ scattering, e.g., we have

$$T_{fi} = -i\int j_\mu^e(x)\left(-\frac{1}{q^2}\right)j_\mu^p(x)\,d^4x = -i\mathcal{M}(2\pi)^4\delta^{(4)}(p+k-p'-k')$$

$$\mathcal{M} \equiv -\frac{e^2}{q^2}\left(j_\mu^{em}\right)_p\left(j^{em\,\mu}\right)_e = (e\bar{u}_p(p')\gamma_\mu u_p(p))\left(-\frac{e^2}{q^2}\right)(-e\bar{u}_e(k')\gamma^\mu u_e(k))$$

A current-current interaction.

Fermi Theory

Now consider $n \to pe^- \bar{\nu}_e$.
Fermi's crucial insight was to realize that the weak currents could be modelled after electromagnetism:

$$\mathcal{M} = G(\bar{u}_p(p')\gamma_\mu u_n(p))(\bar{u}_e(k')\gamma^\mu u_\nu(k))$$

The observation of $e - p$ capture suggests

$$\mathcal{L}_{\mathrm{Fermi}} = -\frac{G_F}{\sqrt{2}} \left\{ (\bar{\psi}_p \gamma_\mu \psi_n)(\bar{\psi}_e \gamma^\mu \psi_\nu) + h.c. \right\}$$

An interaction with charged weak currents.
A weak neutral current was discovered in 1973.
G_F is the Fermi constant, though $G_F \sim 10^{-5}(\mathrm{GeV})^{-2}$.
Suggests the interaction is mediated by massive, spin-one particles.
Fermi's interaction cannot explain the observation of parity violation.
Nor can it explain the $|\Delta J| = 1$ ("Gamow-Teller") transitions observed in nuclear β-decay.
Some $A \times A$ or $T \times T$ interaction has to be present.
Enter the $V - A$ Law....

[Feynman, Gell-Mann, 1958; Sudarshan and Marshak, 1958]

The V-A Law

A “universal” charged, weak current:

$$\mathcal{L} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\left\{\mathcal{J}^\lambda \mathcal{J}^\dagger_\lambda + \mathcal{J}^\dagger_\lambda \mathcal{J}^\lambda\right\} \quad \text{with} \quad \mathcal{J}_\lambda = j^l{}_\lambda + j^h{}_\lambda$$

For the leptons...

$$j^{l\,\lambda} = \bar{\psi}_e \gamma^\lambda (1-\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\mu} + \bar{\psi}_\tau (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\tau}$$

which describes $\nu_l \to l^-$ and $l^+ \to \bar{\nu}_l$ and asserts the leptons do not mix under the weak interactions.

The “V-A” law is equivalent to a “two-component” neutrino picture.

The interactions of the hadrons (quarks) are much richer.

- The strong interaction is strong!
- The quarks *mix* under the weak interactions. E.g., $K^+ \to \mu^+\nu$ is observed. Recall K^+ is $(u\bar{s})$.

Let us continue to focus on neutron β-decay. Recall n is ddu and p is uud.

Isospin is an approximate symmetry:

$M_n = 939.565\,\text{MeV}$ $M_p = 938.272\,\text{MeV}$ $(M_n - M_p)/M_n \ll 1$.

$n \to pe^-\bar{n}u_e$ occurs because isospin is broken $\Longrightarrow$ large τ_n.

Polarized Neutron β-decay in a V-A Theory

$$d^3\Gamma = \frac{1}{(2\pi)^5 2m_B}\left(\frac{d^3\boldsymbol{p}_p}{2E_p}\frac{d^3\boldsymbol{p}_e}{2E_e}\frac{d^3\boldsymbol{p}_\nu}{2E_\nu}\right)\delta^4(p_n - p_p - p_e - p_\nu)\frac{1}{2}\sum_{spins}|\mathcal{M}|^2$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle[\bar{u}_e(p_e)\gamma_\mu(1-\gamma_5)u_\nu(p_\nu)]$$

$$\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle = \bar{u}_p(p_p)(f_1\gamma^\mu - i\frac{f_2}{M_n}\sigma^{\mu\nu}q_\nu + \frac{f_3}{M_n}q^\mu$$
$$-g_1\gamma^\mu\gamma_5 + i\frac{g_2}{M_n}\sigma^{\mu\nu}\gamma_5 q_\nu - \frac{g_3}{M_n}\gamma_5 q^\mu)u_{\vec{n}}(p_n,P)$$

Note $q = p_n - p_p$ and for baryons with polarization P,
$u_{\vec{n}}(p_n,P) \equiv \left(\frac{1+\gamma_5 \not{P}}{2}\right)u_n(p_n)$

$f_1\ (g_V)$	Fermi or Vector	$g_1\ (g_A)$	Gamow-Teller or Axial Vector
$f_2\ (g_M)$	Weak Magnetism	$g_2\ (g_T)$	Induced Tensor or Weak Electricity
$f_3\ (g_S)$	Induced Scalar	$g_3\ (g_P)$	Induced Pseudoscalar

Since $(M_n - M_p)/M_n \ll 1$, a “recoil” expansion is efficacious.
To see how, consider the observables....

Correlation Coefficients

$$d^3\Gamma \propto E_e|\boldsymbol{p}_e|(E_e^{\rm max} - E_e)^2 \times$$
$$[1 + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e E_\nu} + \boldsymbol{P} \cdot (A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e \times \boldsymbol{p}_\nu}{E_e E_\nu})]dE_e d\Omega_e d\Omega_\nu$$

A and B are P odd, T even, whereas D is (pseudo)T odd, P even.
$\lambda \equiv |g_1/f_1| > 0$ and predictions:

$$a = \frac{1-\lambda^2}{1+3\lambda^2} \qquad A = 2\frac{\lambda(1-\lambda)}{1+3\lambda^2} \qquad B = 2\frac{\lambda(1+\lambda)}{1+3\lambda^2} \qquad [+\mathcal{O}(R)]$$

implying $1 + A - B - a = 0$ and $aB - A - A^2 = 0$, testing the V-A structure of the SM to recoil order, $\mathcal{O}(R)$, $R \sim E_e^{\rm max}/M_n \sim 0.0014$.
Currently

$$a = -0.103 \pm 0.004 \quad A = -0.1173 \pm 0.0013 \ (S = 2.3) \quad B = 0.9807 \pm 0.0030$$

so that the relations are satisfied.
With $\tau_n = 885.7 \pm 0.8\,\text{sec}$ and $\tau_n \propto f_1^2 + 3g_1^2$ more tests are possible.

Amsler et al., Particle Data Group, PL B667, 1 (2008) and 2009 partial update http://pdg.lbl.gov.

on the D-term

D can be generated by the SM through electromagnetic final-state interactions. The $\mathcal{O}(\alpha)$ correction vanishes in the zero recoil limit and is estimated to be $D_{\rm SM} \approx 10^{-5}$.

[Callan and Treiman, Phys. Rev. 162, 1494 (1967).]

Recently this calculation has been updated to employ the techqiues of heavy-baryon effective field theory. [Ando, McGovern, Sato, Phys. Lett. B677, 109 (2009).]

The cancellation found in the zero recoil limit has been shown to persist to all orders in α.

There are two expansion parameters: $\alpha/2\pi$ and $\bar{Q}/M_N$, $\bar{Q} \sim m_n - m_p - m_e$.

The $\mathcal{O}((\bar{Q}/M_n)^2)$ term does not contribute.

In $\mathcal{O}(\alpha\bar{Q}/M_N)$ Ando et al. reproduce the Callan-Treiman result and include the leading ($(1/m_\pi)$-enhanced) piece of the N^{3}LO correction to find

$$D = (0.228(p_e^{\rm max}/p_e) + 1.083(p_e/p_e^{\rm max})) \times 10^{-5} - 5.88(p_e^{\rm max}/p_e) \times 10^{-8}$$

with an estimated accuracy of better than 1%.

Experimentally

$D = [-0.6 \pm 1.2(stat) \pm 0.5(syst)] \times 10^{-3}$ [Lising et al., EMIT, Phys. Rev. C 62, 055501 (2000).]

$D = [-2.8 \pm 6.4(stat) \pm 3.0(syst)] \times 10^{-4}$ [Soldner et al., Trine, Phys. Lett. B581, 49 (2004).]

Symmetries of the Hadronic, Weak Current

The values of the 6 couplings (assuming T invariance) are constrained by symmetry.

- Conserved-Vector Current ("CVC") Hypothesis
- Absence of Second-Class Currents ("SCC")
- Partially Conserved Axial Current ("PCAC") Hypothesis

CVC:
The charged weak current and isovector electromagnetic current form an isospin triplet. [Feynman and Gell-Mann, 1958]

$$J_\mu^{em,q} = \frac{2}{3}\bar{\psi}_u \gamma^\mu \psi_u - \frac{1}{3}\bar{\psi}_d \gamma^\mu \psi_d$$

$$J_\mu^{em,q} = e_0 \bar{\psi}_q \gamma^\mu I \psi_q + e_1 \bar{\psi}_q \gamma^\mu \tau_3 \psi_q \quad \text{with} \quad \psi_q = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$\tau_3 \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} = \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} \quad ; \quad \tau_3 \begin{pmatrix} 0 \\ \psi_d \end{pmatrix} = -\begin{pmatrix} 0 \\ \psi_d \end{pmatrix} \quad ; \quad e_{0,1} = \frac{1}{2}(e_u \pm e_d)$$

Symmetries of the Hadronic, Weak Current

Thus

$$J_\mu^{em\,N} = \bar{\psi}[F_1^S(q^2)\gamma^\mu - i\frac{F_2^S(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^S(q^2)}{M_n}q^\mu]e_0 I\psi$$

$$+\bar{\psi}[F_1^V(q^2)\gamma^\mu - i\frac{F_2^V(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^V(q^2)}{M_n}q^\mu]e_1\tau_3\psi$$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \text{and} \quad \tau_+ \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} = \begin{pmatrix} \psi_p \\ 0 \end{pmatrix}$$

The CVC hypothesis implies

$$f_1(q^2) = F_1^V(q^2) \quad \text{and} \quad f_1(q^2) \to 1 \quad \text{as } q^2 \to 0$$
$$f_2(q^2) = F_2^V(q^2)$$
$$f_3(q^2) = F_3^V(q^2) = 0 \quad \text{(current conservation)}$$

$f_1(0) = (1 + \Delta_R^V)V_{ud}$ Δ_R^V starts in $\mathcal{O}(\alpha)$!

[tested to $\mathcal{O}(0.3\%)$ in $0^+ \to 0^+$ decays]

$f_2(0)/f_1(0) = (\kappa_p - \kappa_n)/2 \approx 1.8529$

[tested to $\mathcal{O}(10\%)$ in $A = 12$ system]

The Ademollo-Gatto theorem makes the second test more interesting.

Symmetries of the Hadronic, Weak Current

SCC: "Wrong" G-parity interactions do not appear if isospin is an exact symmetry.
$G \equiv C \exp(i\pi T_2)$ where T_2 is a rotation about the 2-axis in isospin space.

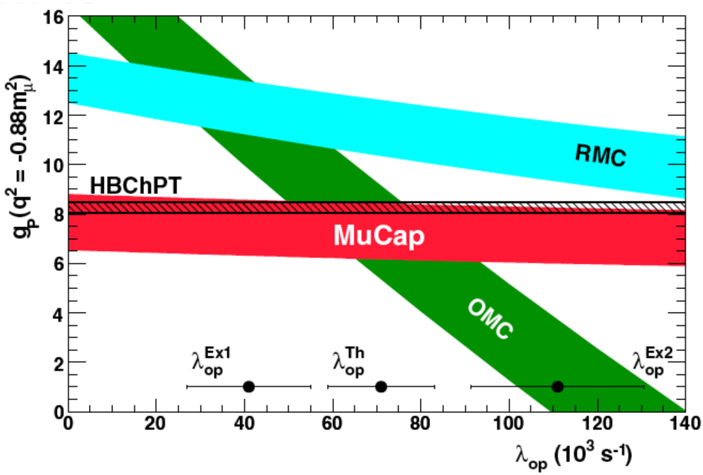
$$\exp(i\pi T_2)\psi = -i\tau_2\psi = \begin{pmatrix} -\psi_n \\ \psi_p \end{pmatrix}$$

$$GV_\mu^{(I)}G^\dagger = +V_\mu^{(I)} \quad ; \quad GA_\mu^{(I)}G^\dagger = -A_\mu^{(I)} \quad \text{"first class"}$$
$$GV_\mu^{(II)}G^\dagger = -V_\mu^{(II)} \quad ; \quad GA_\mu^{(II)}G^\dagger = +A_\mu^{(II)} \quad \text{"second class"}$$

no SCC: $g_2 = 0$ and $f_3 = 0$
(tested to $\mathcal{O}(10\%)$ in $A = 12$ system (combined CVC/SCC test))
PCAC: g_1/f_1 is set by strong-interaction physics:
Goldberger-Treiman relation $\frac{g_1(0)}{f_1(0)} = g_{\pi NN}\frac{f_\pi}{M_N}$
Can test some of these relationships through experiments sensitive to recoil-order effects.

g_3 is also predicted by PCAC (HBChPT) and can be studied in μ capture.
After much controversy, there has been significant progress:

[Andreev et al., MuCap, PRL 99, 032002 (2007).]



Correlation Coefficients in Recoil Order

Consider a and A in recoil order for CVC test. [cf. CVC test in mass 12]
Define $x = \frac{E_l}{E_l^{max}}$ $[0 \leq x \leq 1]$, $\epsilon = (\frac{M_e}{M_n})^2$,
and $R = \frac{E_l^{max}}{m_B} = \frac{M_n^2+M_e^2-M_p^2}{2M_n^2} \sim 0.0014$ (note $\frac{\epsilon}{R} \sim 2.2 \cdot 10^{-4}$) to yield (here $\lambda \equiv g_1/f_1$ and $\tilde{f}_2 \equiv f_2(0)/f_1(0)$, e.g.)

$$
\begin{aligned}
a = \frac{1-\lambda^2}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\Bigg\{ \frac{\epsilon}{Rx}\Big[(1-\lambda^2)(1+2\lambda+\lambda^2 \\
+2\lambda\tilde{g}_2 + 4\lambda\tilde{f}_2 - 2\tilde{f}_3)\Big] + 4R\Big[(1+\lambda^2)(\lambda^2+\lambda \\
+2\lambda(\tilde{f}_2+\tilde{g}_2))\Big] - Rx\Big[3(1+3\lambda^2)^2 + 8\lambda(1+\lambda^2) \\
\times(1+2\tilde{f}_2) + 3(\lambda^2-1)^2\beta^2\cos^2\theta\Big]\Bigg\} + \mathcal{O}(R^2,\epsilon)
\end{aligned}
$$

Correlation Coefficients in Recoil Order

$$A = \frac{2\lambda(1-\lambda)}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\left\{ \frac{\epsilon}{Rx}\Big[4\lambda^2(1-\lambda)(1+\lambda \right.$$
$$+2\tilde{f}_2) + 4\lambda(1-\lambda)(\lambda\tilde{g}_2 - \tilde{f}_3)\Big] + R\Big[\frac{2}{3}(1+\lambda$$
$$+2(\tilde{f}_2+\tilde{g}_2))(3\lambda^2+2\lambda-1)\Big] + Rx\Big[\frac{2}{3}(1+\lambda+2\tilde{f}_2)$$
$$\left. \times(1-5\lambda-9\lambda^2-3\lambda^3) + \frac{4}{3}\tilde{g}_2(1+\lambda+3\lambda^2+3\lambda^3)\Big] \right\}$$
$$+\mathcal{O}(R^2,\epsilon)\ .$$

[Gardner, Zhang, 2001; Bilen'kii et al., 1960; Holstein, 1974]

Coefficients of Rx in A and a yield independent determinations of f_2 and g_2.

[Gardner, Zhang 2001]

Were a and A both measured to $\mathcal{O}(0.1)\%$ (using Rx terms), then $\delta\tilde{f}_2$ is 2.5% and $\delta\tilde{g}_2$ is roughly $0.22\lambda/2$, yielding errors comparable to the mass 12 test

cf. $A = 12$ result $-0.02\lambda \le 2\tilde{g}_2 \le 0.31\lambda$ at 90% C.L. (CVC) [Minamisono et al., 2002]

Uses axial charge difference (th.) $\Delta y = 0.10 \pm 0.05$!

Beyond "V-A" in Neutron β-Decay

The search for non-V-A interactions continues...

$$\mathcal{H}_{int} = (\bar\psi_p \psi_n)(C_S \bar\psi_e \psi_\nu + C'_S \bar\psi_e \gamma_5 \psi_\nu) + (\bar\psi_p \gamma_\mu \psi_n)(C_V \bar\psi_e \gamma^\mu \psi_\nu + C'_V \bar\psi_e \gamma^\mu \gamma_5 \psi_\nu)$$
$$-(\bar\psi_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar\psi_e \gamma^\mu \gamma_5 \psi_\nu + C'_A \bar\psi_e \gamma^\mu \psi_\nu) + (\bar\psi_p \gamma_5 \gamma_\mu \psi_n)(C_P \bar\psi_e \gamma_5 \psi_\nu + C'_P \bar\psi_e \psi_\nu)$$
$$+\frac{1}{2}(\bar\psi_p \sigma_{\lambda\mu} \psi_n)(C_T \bar\psi_e \sigma^{\lambda\mu} \psi_\nu + C'_T \bar\psi_e \sigma^{\lambda\mu} \gamma_5 \psi_\nu) + h.c.$$

[Lee and Yang, 1956; note also Gamow and Teller, 1936]

C'_X denote parity-nonconserving interactions.

In polarized neutron (nuclear) β-decay one more correlation appears: b

$$d^3\Gamma = \frac{1}{(2\pi)^5} \xi E_e |\boldsymbol{p}_e| (E_e^{\max} - E_e)^2 \times$$
$$[1 + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} + \boldsymbol{P} \cdot (A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e \times \boldsymbol{p}_\nu}{E_e E_\nu})] dE_e d\Omega_e d\Omega_\nu$$

[Jackson, Treiman, and Wyld, Phys. Rev. 106, 517 (1957)]

Note, e.g.,

$$b\xi = \pm 2\mathrm{Re}[C_S C_V^* + C'_S C'^*_V + 3(C_T C_A^* + C'_T C'^*_A)]$$

If the electron polarization is also detected, more correlations enter.

Limits from Nuclear β-Decay

Recent limits on b come from nuclear β-decay:

$b = -0.0027 \pm 0.0029$
from survey of $0^+ \to 0^+$ ("superallowed" Fermi) transitions in nuclei

[Towner and Hardy, J. Phys. G, 2003]

$\tilde{a} \equiv a/(1 + bm_e/\langle E_e \rangle) = 0.9981 \pm 0.0030 \pm 0.0037$
from $0^+ \to 0^+$ pure Fermi decay of ${}^{38m}K$

[A. Gorelov et al. PRL 94, 142501 (2005)]

Both limits are consistent with the Standard Model.

Nuclear β-decay spin-isospin selection rules are dictated by the form of the nonrelativistic transition operator.

$$\sum_{j=1}^{A} \tau_\pm(j) = T_\pm \quad \text{"Fermi"} \Longrightarrow J_f = J_i, T_f = T_i \neq 0$$

$$\sum_{j=1}^{A} \sigma(j) \cdot \tau_\pm(j) \quad \text{"Gamow-Teller"} \Longrightarrow \Delta J = 0, 1 \; (J_i = J_f \neq 0)\,,$$

$$\Delta T = 0, 1 \; (T_i = T_f \neq 0)$$

What of the "Two-Component Neutrino"?

We have direct empirical evidence from terrestrial experiments for physics beyond the Standard Model.

Empirical observation of neutrino oscillations [KamLAND, PRL 94, 081801 (2005)] allows us to conclude $\Delta m^2 \equiv m_i^2 - m_j^2 \neq 0$ with surety.

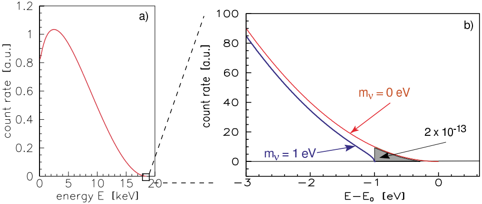
That is, neutrinos have mass.

We see then that the particle content of the Standard Model is incomplete: there is a ν_R, which is "sterile" under Standard Model interactions.

This is not to say that the effects of neutrino mass are large.

Distortions in the shape of the electron energy spectrum in ^{3}H β-decay near its endpoint bound m_ν^2. [KATRIN, loi]

The neutrino mass is still "practically" zero, but the two-component neutrino picture may fail – the neutrino may not be its own antiparticle!



Bibliography

The following textbooks provide helpful background and should aid in assimilating the information found in the journal articles cited throughout.

- Hoddeson, Brown, Riordan, and Dresden, eds., *The Rise of the Standard Model*, 1997
- Bigi and Sanda, *CP Violation*, 2000
- Branco, Lavoura, and Silva, *CP Violation*, 1999
- Cheng and Li, *Gauge Theory of Elementary Particle Physics*, 1984
- Commins and Bucksbaum, *Weak Interactions of Leptons and Quarks*, 1983.
- Halzen and Martin, *Quarks and Leptons: An Introduction Course in Modern Particle Physics*, 1984.
- Holstein, *Weak Interactions in Nuclei*, 1989
- Khriplovich and Lamoreaux, *CP Violation Without Strangeness*, 1997
- Peskin and Schroeder, *An Introduction to Quantum Field Theory*, 1995
- Sachs, *The Physics of Time Reversal*, 1987