Hybrid Variance Reduction Method Based on Gaussian Process

Consortium for Advanced Simulation of Light Water Reactors

Zeyun Wu, Qiong Zhang, and Hany S. Abdel-Khalik
Department of Nuclear Engineering, North Carolina State University

INTRODUCTION

Variance reduction techniques are employed to accelerate the convergence of Monte Carlo (MC) simulation. Hybrid deterministic-MC methods utilize deterministic models (both forward and adjoint) to bias source particles and assign appropriate importance map to MC models. If done properly, hybrid methods have been shown to accelerate the convergence of MC simulation - that is obtaining acceptable reduction in the statistical uncertainties for the responses of interest with less computational overhead.

Under the assumption that the statistical uncertainties resulting from the radiation transport may be treated as a Gaussian Process[1], a new hybrid method for global variance reduction (GVR) is presented. The principle behind this method is recalled here along with a comparison to FW-CADIS method[2], which is currently implemented in the SCALE package. The construction of the covariance matrix as needed by the GP method is implemented in the MAVRIC sequence of SCALE to facilitate the comparison of the GP and the FW-CADIS methods. In this work the GP method is applied two study cases - a simplified BWR assembly model and a higher dimensional problem representing a prototypical PWR core model – to demonstrate the efficiency of this method.

METHODOLOGY

GP theory assumes that random fluctuations of a response follow a process that can be described by a normal distribution. If the statistical uncertainties for all responses of interest (say the group fluxes everywhere in the domain) can be treated as GPs, one could describe the correlations between these uncertainties via a covariance matrix. If this matrix is dense and ill-conditioned as most of practical physics problems yield, one could in principle reduce the computational overhead required to reduce the uncertainties everywhere by taking advantage of their correlations. The lower the effective rank of the covariance matrix, the more correlations between the responses, and the lower the computational overhead needed to reduce the variances globally.

Let $\vec{r} = [r_1 \quad r_2 \quad \dots \quad r_n]^T \in \mathbf{R}^n$ represent a vector of the *n* responses of interest representing n random Gaussian processes. Denote $\{r_i^k\}_{i=1}^n, k = 1, 2, \dots, N$ as *N* realizations of these random processes. The covariance between the two responses r_i and r_j is given by:

$$\mathrm{cov}(r_i, r_j) = \lim_{N\to\infty} \frac{1}{N-1} \sum_{k=1}^{N} \left(r_i^k - \hat{r}_i\right)\left(r_j^k - r_j\right).$$

The covariance information between all pairs of n responses may also be represented by a symmetric covariance matrix $\mathbf{C} \in \mathbf{R}^{n\times n}$ such that: $\mathbf{C}_{ij} = \mathrm{cov}(r_i, r_j)$. The singular value decomposition (SVD) form of this matrix is:

$$\mathbf{C} = \mathbf{W}\boldsymbol{\Sigma}\mathbf{W}^T = \sum_{i=1}^{r} \sigma_i \vec{w}_i \vec{w}_i^T.$$

To provide a basis for comparison, we contrast the GP method to the FW-CADIS method[2]. In both of these methods, the weight windows-based biasing is determined by solving an adjoint equation of the form (demonstrated first for a single response) :

$$\mathbf{L}^*\left(\vec{\phi}_i^*\right) = \frac{\partial r_i}{\partial \vec{\phi}}.$$

To achieve GVR, the FW-CADIS method formulates a single pseudo response of the form:

$$\xi^{\text{FW-CADIS}} = \sum_{i=1}^{n} \frac{1}{\phi_i} r_i.$$

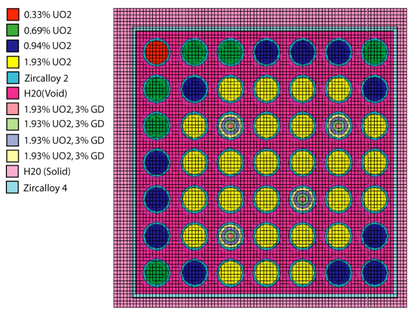
In this method, more MC particles are sent to regions in the phase space where the flux is low. In contrast, the GP method forms pseudo response as follows:

$$\xi_j^{\text{GP}} = \vec{w}_j^T \vec{r}, \quad j = 1, 2, \dots, r.$$

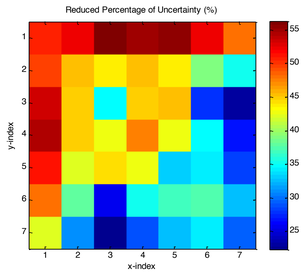
They are calculated from the singular vectors of the covariance matrix and executed based on the effective subspace of the matrix rather than the whole space to save computational overheads.

CASE STUDY #1: BWR Assembly Model

The simplified BWR assembly model is developed with MAVRIC sequences in SCALE package (see right figure). It consists of 7x7 array of fuel pins with various enrichments. A fixed source subcritical system is presented and a data library with total 27 neutron and 19 photon energy group is applied.



The relative uncertainty comparison of spatial thermal flux for different methods are depicted as the bar plots in the figures below; the metric employed for the comparison is presented on the right.



GP vs. FW-CADIS

Pseudo Response

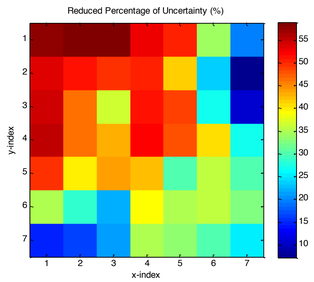
- FW-CADIS:

$$\xi = \sum_{i=1}^{m} r_i / \phi_i$$

- GP:

$$\xi_j = \vec{w}_j^T \vec{r}, \quad j = 1, \cdots, r$$

$$\Delta\sigma_i = \frac{\sigma_i^{\text{GP}} - \sigma_i^{\text{FW-CADIS}}}{\sigma_i^{\text{FW-CADIS}}} \times 100\%$$



Coupled GP+FW-CADIS vs. FW-CADIS

Pseudo Response

- FW-CADIS:

$$\xi = \sum_{i=1}^{m} r_i / \phi_i$$

- Coupled GP + FW-CADIS:

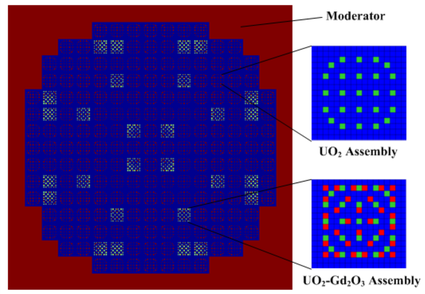
$$\psi_j = \vec{w}_j^T \vec{\phi}, \quad j = 1, \cdots, r$$

$$\xi_j = \vec{w}_j^T \vec{r}, \quad j = 1, \cdots, r$$

$$\xi_{\text{com}} = \sum_{j=1}^{r} \frac{1}{\psi_j} \sigma_j \xi_j$$

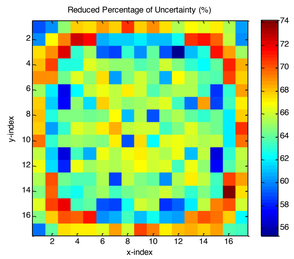
$$\Delta\sigma_i = \frac{\sigma_i^{\text{Coupled}} - \sigma_i^{\text{FW-CADIS}}}{\sigma_i^{\text{FW-CADIS}}} \times 100\%$$

CASE STUDY #2: PWR Core Model

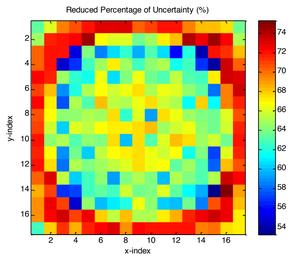


The left figure is the X-Y view of the core loading pattern with details assembly described on the side, There are total 193 fuel assemblies (blue region) laid out a 17x17 grid scheme and surrounded by light water (red region) and two types of fuel assemblies are designed: UO_2 fuel assembly and a UO_2-Gd_2O_3 fuel assembly.

The relative uncertainty comparison of thermal flux for different methods are presented in figures as follows:



GP vs. FW-CADIS



Coupled GP+FW-CADIS vs. FW-CADIS

CONCLUSIONS

- GP provides one approach to take advantage of responses uncertainties correlations
- Number of independent correlations are much smaller than number of responses, when responses are required everywhere in phase space
- Deterministic models could be employed to identify correlations
- Idea could be extended to other hybrid deterministic-MC techniques and to find correlations between responses sensitivities and eigenvalue sensitivities

REFERENCES

[1] M. KENNEDY and A. O'HAGAN, "Bayesian calibration of computer model," *Journal of the Royal Statistical Society*, 63, 3, 425–464 (2001).

[2] J. C. WAGNER, E. D. BLAKEMAN, and D. E. PEPLOW, "A Forward-Weighted CADIS Method for Global Variance Reduction," *Trans. Am. Nucl. Soc. Meeting*, Washington, D.C., 97 (Nov. 2007).

CONTACT INFORMATION

NC STATE UNIVERSITY
Department of NUCLEAR ENGINEERING

Zeyun Wu
zwu3@ncsu.edu
Hany S. Abdel Khalik
abdelkhalik@ncsu.edu