The basic equations of Neutron Spin-Echo

CHRNS — Center for High Resolution Neutron Scattering — NSF — NIST

In an NSE experiment, a polarized neutron beam with a wavelength spread of ~15% travels through two magnetic fields of (almost) equal intensity. At the end of the first magnetic field, the spin state of each neutron is reversed; then the neutron beam interacts with the sample changing its direction and, possibly, its energy; finally it goes through the second magnetic field.

$$\varphi = \gamma \frac{\int B dl}{v}$$

The precession angle of the spin of a neutron traveling in a magnetic field is proportional to the field path, $J = \int B dl$.

The polarization of a neutron beam with a velocity distribution $f(v)$ traveling in a magnetic field:

$$P_x = \langle \cos\varphi \rangle = \int f(v) \cos\left[\frac{\gamma \int B dl}{v}\right] dv$$

Elastic scattering

$$\overline{\varphi} = \left\langle \gamma \frac{\int B_0 dl}{v} - \gamma \frac{\int B_1 dl}{v} \right\rangle_{f(\lambda)}$$

Echo Condition: $J_0 = J_1 \quad \overline{\varphi} = 0 \quad P_x = 1$

Note: The requirement that φ=0 is can be in some cases released. This treatment is valid for the most common case of Quasi-Elastic Scattering

Quasi-Elastic scattering

$$\varphi = \left\langle \gamma \frac{\int B_0 dl}{v(\lambda)} - \gamma \frac{\int B_1 dl}{v(\lambda) + \delta v} \right\rangle$$

Series Expansion in δλ and δJ

$$\varphi \approx \gamma \frac{m}{h} J_0 \delta\lambda + \gamma \frac{m}{h} (J_0 - J_1)\lambda$$

$$\hbar\omega = \Delta E = \frac{h^2}{2m}\left[\frac{1}{\lambda^2} - \frac{1}{(\lambda + \delta\lambda)^2}\right] \approx \frac{h^2}{m}\frac{\delta\lambda}{\lambda^3} \qquad \delta\lambda = \frac{\omega}{2\pi h} m \lambda^3$$

$$\varphi = \gamma \frac{m^2 \lambda^3}{2\pi h^2} J_0 \omega + \gamma \frac{m}{h} \cancel{(J_0 - J_1)} \lambda$$

0 at the echo condition

In an NSE measurement the experimentally determined quantity is the polarization of the scattered beam. This quantity is given by a double average on both incoming beam wavelength distribution and on the dynamic structure factor, $S(Q,\omega)$, which represents the probability that the scattered neutron has exchanged an energy equal to $\hbar\omega$ with the sample.

$$P_x = \langle \cos(\varphi) \rangle = \int\int f(\lambda) S(Q,\omega) \cos\left[\gamma \frac{m^2 \lambda^3}{2\pi h^2} J_0 \omega + \gamma \frac{m}{h} (J_0 - J_1) \lambda\right] d\lambda d\omega$$

Assuming Quasielastic scattering, the dynamic structure factor is an even function, the integral can be factorized

$$P_x = \langle \cos(\varphi) \rangle = \int f(\lambda) \cos\left[\gamma \frac{m}{h} (J_0 - J_1)\lambda\right] d\lambda \times \int S(Q,\omega) \cos\left[\gamma \frac{m^2 \lambda^3}{2\pi h^2} J_0 \omega\right] d\omega$$

Fourier time

$$t = \gamma \frac{m^2 \lambda^3}{2\pi h^2} J_0$$

The polarization is given by the product of two integrals. The first Integral represents the Fourier transform of the wavelength distribution, where the conjugated variable is the difference in field path. The difference in the field path is indicated as an angle, and is called phase, ph.

The second integral represents the Fourier transform of the dynamic structure factor, $S(Q,\omega)$, where the conjugated variable is $t = (\gamma m^2 \lambda^3 J_0)/(2\pi h^2)$. The Fourier transform of $S(Q,\omega)$ is the Intermediate scattering function. The meaning of the equation is that measuring the polarization of the scattered beam, as a function of the field integral, J_0, we effectively measure the intermediate scattering function. The quantity is the so called Fourier time, t. The Fourier time is in a sense a technical quantity, by the experimental conditions (λ and J_0), but being the conjugated variable of the energy, it has also a physical meaning and represents a real physical time.

$$P_x\left(\Delta J^{ph_i}, Q, t\right) = P_s\left(\Delta J^{ph_i}\right) \frac{\int S(Q,\omega) \cos[\omega t] d\omega}{\int S(Q,\omega) d\omega}$$

Through the polarization, NSE measures the Fourier Transform of the Dynamic Structure Factor